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DAMAGE IDENTIFICATION OF SUBSTRUCTURE USING AN ADAPTIVE TRACKING TECHNIQUE

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ABSTRACT

A challenging problem in structural system identification and damage detection lies in the requirement of a large number of sensors and the numerical difficulty in obtaining reasonably accurate results when the system is large. To address this issue, the substructure identification (SSI) approach has been developed based on measured response data and external excitations. Due to practical limitations, the response data may not be available at all degrees of freedom of the structure and that the external excitations may not be measured (or available). In this paper, a new data analysis method, referred to as the sequential nonlinear least-square estimation with unknown inputs and unknown outputs (SNLSE-UI-UO) along with the sub-structure approach will be used to identify damages at critical locations of the complex structure. In our approach, only a limited number of response data are needed and the external excitations may not be measured. The accuracy of this approach is demonstrated using a long-span truss with finite-element formulation and an 8-story base-isolated building. Simulation results demonstrate that the proposed approach is capable of tracking the changes of structural parameters, leading to the identification of structural damages at critical locations.

Keywords: System Identification and Damage Detection, Substructure Identification, Structural Health Monitoring

INTRODUCTION

The development of a health monitoring system to ensure the reliability and safety of structures has received considerable attention recently. In particular, the ability to detect structural damage, based on measured vibration data, is of practical importance. Various analysis methodologies for structural damage identification have been proposed [e.g., Bernal & Beck (2004), Lin et al (2005)]. However, most of the methodologies available in the literature [e.g., Bernal & Beck (2004), Lin et al (2005)] deal with linear structures and require both the reference data (the data without damage) and the data after damage. In practice, however, the reference data may not be available or difficult to establish, and after a severe event, such as a strong earthquake, it may not be feasible to conduct vibration tests to obtain meaningful data for damage identification. It would be desirable for a data analysis method to be capable of detecting the structural damage based solely on the vibration data measured during a severe event, such as a strong earthquake, without a prior knowledge of the undamaged structure. In this connection, several damage identification methodologies have been developed recently, including the least-square estimation (LSE) [e.g., Lin, et al (2001), Yang and Lin (2004a, 2005)], the extended Kalman filter (EKF) [e.g., Yang, et al (2006b, c)], the sequential nonlinear least-square estimation [Yang, et al (2006a)], and others [see references in Yang and Lin (2005)].

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In the approaches mentioned above, external excitations (inputs) should be available from sensor measurements. Due to practical limitations, it may not be possible to install enough sensors in the health monitoring system to measure either all the external excitations (inputs) or the acceleration responses (outputs) at all DOFs. In fact, it is highly desirable to install as few sensors as possible. When the external excitations are not measured or not available, analytical recursive solutions with adaptive damage tracking capabilities have been proposed to identify the structural damage based on: (i) the LSE approach [Yang, et al (2004b; 2007)], and (ii) the extended Kalman filter technique [Yang, et al (2006c)]. Recently, a new technique, referred to as the sequential nonlinear least squares estimation with unknown inputs (excitations) and unknown outputs (responses) (SNLSE-UI-UO), has been developed [Yang and Huang (2006d)]. In this approach, external excitations and some acceleration responses are not needed, so that the number of sensors required in the health monitoring system can be reduced. This new technique is capable of tracking the variations of structural parameters, such as the degradation of stiffness, due to damages.

In practical applications, the modeling of engineering structures often involves a large number of degrees of freedom (DOFs), leading to not only numerical difficulties for an accurate damage detection, but also the requirement of excessive number of sensors. It is highly desirable to reduce the required number of sensors as much as possible due to economic considerations and data management. Further, for a complex structure, there may only be a limited number of hot spots or critical areas where damages may likely to occur, and hence the health monitoring can be restricted to such critical areas. This will allow for a significant reduction of the number of required sensors. Consequently, structures can be decomposed into smaller subsystems for the purpose of damage identification. In this connection, the so-called substructure identification (SSI) approach [e.g. Koh et al (1991, 2003)] has been investigated.

In this paper, we present the application of the newly developed SNLSE-UI-UO method and the sub-structure approach to identify sub-structural damages in complex structures based on limited number of measured vibration data and the finite-element formulation. We shall demonstrate the feasibility of the local health monitoring for critical elements without the global information of the structure, thus reducing the required number of sensors and the burden of data management, such as the data transmission and analyses. Simulation results using a long-span truss and an 8-story base-isolated building will be presented to demonstrate the accuracy of the proposed approach in tracking the variation of sub-structural parameters due to damages.

SEQUENTIAL NONLINEAR LSE WITH UNKNOWN INPUTS AND UNKNOWN OUTPUTS

Let $\mathbf{x} = [x_1(t), x_2(t), ..., x_m(t)]^T$ and $\dot{\mathbf{x}} = [\dot{x}_1(t), \dot{x}_2(t), ..., \dot{x}_m(t)]^T$ be the displacement and velocity vectors, respectively, of a m-DOF nonlinear structure to be considered. The acceleration vector $\ddot{\mathbf{x}}(t) = [\ddot{x}_1(t), \ddot{x}_2(t), ..., \ddot{x}_m(t)]^T$ is divided into two vectors, denoted by $\ddot{\mathbf{x}}^*(t) = [\ddot{x}_1^*(t), \ddot{x}_2^*(t), ..., \ddot{x}_s^*(t)]^T$ and $\ddot{\bar{\mathbf{x}}}(t) = [\ddot{\bar{x}}_1(t), \ddot{\bar{x}}_2(t), ..., \ddot{\bar{x}}_{m-s}(t)]^T$, in which $\ddot{x}_i^*(t)$ (i = 1, 2, …, s) and $\ddot{\bar{x}}_i(t)$ (i = 1, 2, …, m-s) are unknown (unmeasured) and known (measured) acceleration responses (outputs), respectively. In a similar manner, the external excitations are divided into two vectors, $\mathbf{f}^*(t) = [f_1^*(t), f_2^*(t), ..., f_r^*(t)]^T$ and $\mathbf{f}(t) = [f_1(t), f_2(t), ..., f_{\overline{m}}(t)]^T$, where $f_i^*(t)$ (i = 1, 2, …, r) and $f_i(t)$ (i = 1, 2, …, $\overline{m}$) are unknown (unmeasured) and known (measured) excitations (inputs), respectively. The equation of motion of the m-DOF nonlinear structure can be expressed as

$$\overline{\mathbf{M}}\,\ddot{\bar{\mathbf{x}}}(t) + \mathbf{F}_c[\dot{\mathbf{x}}(t)] + \mathbf{F}_s[\mathbf{x}(t)] = \boldsymbol{\eta}^*\mathbf{f}^*(t) + \boldsymbol{\eta}\,\mathbf{f}(t) - \mathbf{M}^*\,\ddot{\mathbf{x}}^*(t) \qquad (1)$$

in which $\overline{\mathbf{M}}$ = [$m \times (m-s)$] mass matrix corresponding to the (m–s)-known (measured) acceleration response vector $\ddot{\bar{\mathbf{x}}}(t)$; $\mathbf{M}^*$ = ($m \times s$) mass matrix corresponding to the s-unknown (unmeasured) acceleration response vector $\ddot{\mathbf{x}}^*(t)$ (or unknown outputs); $\mathbf{F}_c[\dot{\mathbf{x}}(t)]$ = m-damping force vector; $\mathbf{F}_s[\mathbf{x}(t)]$ = m-stiffness force vector; $\mathbf{f}(t)$ = $\overline{m}$ -known (measured) excitation vector; $\boldsymbol{\eta}$ = $m \times \overline{m}$ excitation influence matrix corresponding to $\mathbf{f}(t)$; $\mathbf{f}^*(t)$ = r-unknown (unmeasured) excitation vector (or unknown inputs); and $\boldsymbol{\eta}^*$ = $m \times r$ excitation influence matrix corresponding to $\mathbf{f}^*(t)$. For simplicity of presentation, the argument t of all quantities above will be dropped in the following. Further, the bold-face letter represents either a vector or a matrix.

The unknown quantities to be identified are the unknown excitation (input) vector $\mathbf{f}^*$, the unmeasured acceleration response (output) vector $\ddot{\mathbf{x}}^*$, the state vector $\mathbf{X} = [\mathbf{x}^T, \dot{\mathbf{x}}^T]^T$, including the displacement and velocity vectors, and the parametric vector $\boldsymbol{\theta} = [\theta_1, \theta_2, ..., \theta_n]^T$ of the structure, involving n unknown parameters, θ_i (i = 1, 2, … , n), such as stiffness, damping and nonlinear parameters. Our objective is to determine not only all the unknown quantities above but also the variation of the parametric vector $\boldsymbol{\theta}$ due to structural damages, such as the degradation of stiffness, etc. Hence, $\boldsymbol{\theta}$ will be treated as a time varying function later. For simplicity of derivation, $\boldsymbol{\theta}$ will first be considered as a constant vector in this section, and the solution thus obtained will be extended in the next section using the adaptive tracking technique to account for the variation of $\boldsymbol{\theta}$ as a function of time.

The observation equation associated with the equation of motion, Eq.(1), can be written as

$$\boldsymbol{\varphi}(\mathbf{X})\boldsymbol{\theta} + \boldsymbol{\varepsilon} = \overline{\boldsymbol{\eta}}\,\overline{\mathbf{f}} + \mathbf{y} \tag{2}$$

where $\boldsymbol{\varphi}(\mathbf{X})$ is the observation matrix, $\mathbf{X}$ is the state vector defined above, $\mathbf{y} = \boldsymbol{\eta}\mathbf{f} - \overline{\mathbf{M}}\,\ddot{\overline{\mathbf{x}}}$ is known, $\overline{\mathbf{f}} = [\mathbf{f}^{*T} \quad \ddot{\mathbf{x}}^{*T}]^T$ is an unknown vector consisting of unknown inputs $\mathbf{f}^*$ and unknown outputs $\ddot{\mathbf{x}}^*$, $\overline{\boldsymbol{\eta}} = [\boldsymbol{\eta}^* \quad -\mathbf{M}^*]$, and $\boldsymbol{\varepsilon}(t)$ is the model noise. Eq.(2) can be discretized at $t = t_k = k\Delta t$ as

$$\boldsymbol{\varphi}_k(\mathbf{X}_k)\boldsymbol{\theta}_k + \boldsymbol{\varepsilon}_k - \overline{\boldsymbol{\eta}}\,\overline{\mathbf{f}}_k = \mathbf{y}_k \tag{3}$$

in which $\mathbf{X}_k = \mathbf{X}(t_k)$, $\boldsymbol{\varphi}_k(\mathbf{X}_k) = \boldsymbol{\varphi}[\mathbf{X}(t_k); t_k]$, $\boldsymbol{\theta}_k = \boldsymbol{\theta}(t_k)$, $\boldsymbol{\varepsilon}_k = \boldsymbol{\varepsilon}(t_k)$, $\overline{\mathbf{f}}_k = \overline{\mathbf{f}}(t_k)$ and $\mathbf{y}_k = \mathbf{y}(t_k)$.

Define an extended unknown vector $\boldsymbol{\theta}_{e,k}$ at t_k

$$\boldsymbol{\theta}_{e,k} = \begin{bmatrix} \boldsymbol{\theta}_k \\ \overline{\mathbf{f}}_k \end{bmatrix} \tag{4}$$

where $\boldsymbol{\theta}_{e,k}$ is a $(n + r + s)$ -unknown vector. Then, Eq.(3) can be expressed as

$$\boldsymbol{\varphi}_{e,k}(\mathbf{X}_k)\boldsymbol{\theta}_{e,k} + \boldsymbol{\varepsilon}_k = \mathbf{y}_k \tag{5}$$

in which $\boldsymbol{\varphi}_{e,k}(\mathbf{X}_k) = [\boldsymbol{\varphi}_k(\mathbf{X}_k) \quad -\overline{\boldsymbol{\eta}}]$. Note that $\mathbf{X}_k$ and $\boldsymbol{\theta}_{e,k}$ in Eq.(5) are unknown quantities to be estimated. Hence, Eq.(5) is a nonlinear vector equation with unknowns $\mathbf{X}_k$ and $\boldsymbol{\theta}_{e,k}$.

Instead of solving $\mathbf{X}_k$ and $\boldsymbol{\theta}_{e,k}$ simultaneously by forming an extended composite unknown vector, we propose to solve $\mathbf{X}_k$ and $\boldsymbol{\theta}_{e,k}$ in two steps. The first step is to determine $\boldsymbol{\theta}_{e,k}$ by assuming (or under the condition) that $\mathbf{X}_k$ is given, and the second step is to determine $\mathbf{X}_k$ through a nonlinear LSE approach, referred to as the sequential nonlinear least square estimation with unknown inputs and unknown outputs (SNLSE-UI-UO), as follows.

STEP I: Determination of recursive solutions for $\boldsymbol{\theta}_{k+1}(\mathbf{X}_{k+1})$ and $\bar{\mathbf{f}}_{k+1|k+1}$ given $\mathbf{X}_{k+1}$

Suppose the state vector $\mathbf{X}_{k+1}$ is known and the parametric vector $\boldsymbol{\theta}_k$ is constant, i.e., $\boldsymbol{\theta} = \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 = ... = \boldsymbol{\theta}_k$. Based on Eqs.(3)-(5), the sum squares error can be expressed as

$$J_{k+1}(\boldsymbol{\theta}_{e,k}) = \sum_{i=1}^{k+1} [\mathbf{y}_i - \boldsymbol{\varphi}_{e,i}(\mathbf{X}_i)\boldsymbol{\theta}_{e,i}]^T [\mathbf{y}_i - \boldsymbol{\varphi}_{e,i}(\mathbf{X}_i)\boldsymbol{\theta}_{e,i}] \tag{6}$$

If the number of DOFs, m, of the structure is greater than the total number, $s + r$, of unknown inputs and unknown outputs $\bar{\mathbf{f}} = [\mathbf{f}^{*T} \quad \ddot{\mathbf{x}}^{*T}]^T$, i.e., $m > s + r$, the LSE approach can be used to minimize the objective function given by Eq.(6) to yield the recursive LSE solution for the extended unknown vector $\boldsymbol{\theta}_{e,k+1}$. The recursive solution for the estimates of $\boldsymbol{\theta}_{k+1}$ and $\bar{\mathbf{f}}_{k+1}$, denoted by $\hat{\boldsymbol{\theta}}_{k+1}$ and $\hat{\bar{\mathbf{f}}}_{k+1|k+1}$, respectively, can be obtained as follows [see Yang et al (2004b, 2007), Yang and Huang (2006d)],

$$\hat{\boldsymbol{\theta}}_{k+1}(\mathbf{X}_{k+1}) = \hat{\boldsymbol{\theta}}_k + \mathbf{K}_{\boldsymbol{\theta},k+1}(\mathbf{X}_{k+1})[\mathbf{y}_{k+1} - \boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})\hat{\boldsymbol{\theta}}_k + \bar{\boldsymbol{\eta}}\hat{\bar{\mathbf{f}}}_{k+1|k+1}] \tag{7}$$

$$\hat{\bar{\mathbf{f}}}_{k+1|k+1} = -\mathbf{S}_{k+1}(\mathbf{X}_{k+1})\bar{\boldsymbol{\eta}}^T[\mathbf{I} - \boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})\mathbf{K}_{\boldsymbol{\theta},k+1}(\mathbf{X}_{k+1})][\mathbf{y}_{k+1} - \boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})\hat{\boldsymbol{\theta}}_k] \tag{8}$$

$$\mathbf{K}_{\boldsymbol{\theta},k+1}(\mathbf{X}_{k+1}) = \mathbf{P}_{\boldsymbol{\theta},k}\boldsymbol{\varphi}_{k+1}^T(\mathbf{X}_{k+1})[\mathbf{I} + \boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})\mathbf{P}_{\boldsymbol{\theta},k}\boldsymbol{\varphi}_{k+1}^T(\mathbf{X}_{k+1})]^{-1} \tag{9}$$

$$\mathbf{S}_{k+1}(\mathbf{X}_{k+1}) = \{\bar{\boldsymbol{\eta}}^T[\mathbf{I} - \boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})\mathbf{K}_{\boldsymbol{\theta},k+1}(\mathbf{X}_{k+1})]\bar{\boldsymbol{\eta}}\}^{-1} \tag{10}$$

$$\begin{aligned}\mathbf{P}_{\boldsymbol{\theta},k+1} = {} & [\mathbf{I} + \mathbf{K}_{\boldsymbol{\theta},k+1}(\mathbf{X}_{k+1})\bar{\boldsymbol{\eta}}\mathbf{S}_{k+1}(\mathbf{X}_{k+1})\bar{\boldsymbol{\eta}}^T\boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})] \\ & \bullet[\mathbf{P}_{\boldsymbol{\theta},k} - \mathbf{K}_{\boldsymbol{\theta},k+1}(\mathbf{X}_{k+1})\boldsymbol{\varphi}_{k+1}(\mathbf{X}_{k+1})\mathbf{P}_{\boldsymbol{\theta},k}]\end{aligned} \tag{11}$$

in which $\mathbf{K}_{\boldsymbol{\theta},k+1}(\mathbf{X}_{k+1})$ is the LSE gain matrix.

STEP II: Determination of the estimate $\hat{\mathbf{X}}_{k+1|k+1}$ for $\mathbf{X}_{k+1}$ based on nonlinear LSE

Since $\boldsymbol{\theta}_{k+1}$ and $\mathbf{X}_{k+1}$ are interrelated, the estimate $\hat{\boldsymbol{\theta}}_{k+1}$ is a function of the unknown state vector $\mathbf{X}_{k+1}$, i.e., $\hat{\boldsymbol{\theta}}_{k+1} = \hat{\boldsymbol{\theta}}_{k+1}(\mathbf{X}_{k+1})$, as shown in Eq.(7). It follows from Eq.(6) that the general objective function should be expressed as

$$J_{k+1}(\mathbf{X}_{k+1}) = \sum_{i=1}^{k+1} [\mathbf{y}_i - \boldsymbol{\varphi}_i(\mathbf{X}_i)\hat{\boldsymbol{\theta}}(\mathbf{X}_i) + \bar{\boldsymbol{\eta}}\hat{\bar{\mathbf{f}}}_i]^T [\mathbf{y}_i - \boldsymbol{\varphi}_i(\mathbf{X}_i)\hat{\boldsymbol{\theta}}(\mathbf{X}_i) + \bar{\boldsymbol{\eta}}\hat{\bar{\mathbf{f}}}_i] \tag{12}$$

and the unknown state vector $\mathbf{X}_{k+1}$ will be estimated by further minimizing the general objective function in Eq.(12). Since Eq.(12) is highly nonlinear in unknown state vector $\mathbf{X}_{k+1}$, the non-linear least-square estimation approach proposed in Yang, et al (2006a) is used to estimate $\mathbf{X}_{k+1}$, denoted by $\hat{\mathbf{X}}_{k+1|k+1}$, and the result is given as follows,

$$\hat{\mathbf{X}}_{k+1|k+1} = \hat{\mathbf{X}}_{k+1|k} + \overline{\mathbf{K}}_{k+1}[\mathbf{y}_{k+1} - \hat{\mathbf{y}}_{k+1}(\hat{\mathbf{X}}_{k+1|k})] \tag{13}$$

in which

$$\hat{\mathbf{X}}_{k+1|k} = \boldsymbol{\Phi}_{k+1,k}\hat{\mathbf{X}}_{k|k} + \mathbf{B}_1\ddot{\mathbf{x}}_k + \mathbf{B}_2\ddot{\mathbf{x}}_{k+1} \tag{14}$$

$$\overline{\mathbf{K}}_{k+1} = \overline{\mathbf{P}}_{k+1|k}\boldsymbol{\Psi}_{k+1,k+1}^T[\mathbf{I} + \boldsymbol{\Psi}_{k+1,k+1}\overline{\mathbf{P}}_{k+1|k}\boldsymbol{\Psi}_{k+1,k+1}^T]^{-1} \tag{15}$$

$$\overline{\mathbf{P}}_{k+1|k} = \boldsymbol{\Phi}_{k+1,k}\overline{\mathbf{P}}_{k|k}\boldsymbol{\Phi}_{k+1,k}^T \tag{16}$$

$$\overline{\mathbf{P}}_{k|k} = \overline{\mathbf{P}}_{k|k-1} - \overline{\mathbf{K}}_k\boldsymbol{\Psi}_{k,k}\overline{\mathbf{P}}_{k|k-1} = (\mathbf{I} - \overline{\mathbf{K}}_k\boldsymbol{\Psi}_{k,k})\overline{\mathbf{P}}_{k|k-1} \tag{17}$$

In equations above, $\mathbf{\Phi}_{k+1,k}$ is the transition matrix for the state vector from k to k+1, and $\hat{\mathbf{y}}_{k+1}(\hat{\mathbf{X}}_{k+1|k}) = \mathbf{\varphi}_{k+1}[\mathbf{X}_{k+1}(\hat{\mathbf{X}}_{k+1|k})]\hat{\mathbf{\theta}}_{k+1}[\mathbf{X}_{k+1}(\hat{\mathbf{X}}_{k+1|k})] - \bar{\mathbf{\eta}}\hat{\bar{\mathbf{f}}}_{k+1}$. Thus, the estimate $\hat{\mathbf{X}}_{k+1|k+1}$ obtained from Eqs.(13)-(17) will be used to replace $\mathbf{X}_{k+1}$ in Eqs.(7)-(11) for computing the estimates of unknown parametric vector $\hat{\mathbf{\theta}}_{k+1}$ and unknown inputs and outputs $\hat{\bar{\mathbf{f}}}_{k+1}$. The new methodology proposed and derived above is referred to as the sequential nonlinear least square estimation with unknown inputs and unknown outputs (SNLSE-UI-UO) [Yang and Huang (2006d)]. The analytical solutions derived and presented in Eqs.(7)-(11) and (13)-(17) are not available in the previous literature.

ADAPTIVE TRACKING

The recursive solution $\hat{\mathbf{\theta}}_{k+1}$ in Eqs. (7)-(11) is derived based on the premise of constant parametric vector $\mathbf{\theta}_{k+1}$. Here, we use the adaptive tracking technique proposed in Yang and Lin (2005) to identify time-varying parameters of structures for detecting the damage. To track the variation of each parameter, say the jth element $\theta_j(k+1)$ of $\mathbf{\theta}_{k+1}$, the estimation error $[\theta_j(k) - \hat{\theta}_j(k)]$ is proposed to be expressed by $\lambda_j(k+1)[\theta_j(k) - \hat{\theta}_j(k)]$, where $\lambda_j(k+1)$ will be determined from the current measured data, so that the residual error is contributed only by the noise, eliminating the contribution due to the parametric variation. It can be shown that $\mathbf{P}_{\mathbf{\theta},k}$ in Eq.(9) is proportional to the covariance matrix of the estimation error at t_k, i.e., $\mathbf{P}_{\mathbf{\theta},k} = \sigma^{-2}E[(\mathbf{\theta}_k - \hat{\mathbf{\theta}}_k)(\mathbf{\theta}_k - \hat{\mathbf{\theta}}_k)^T]$, where σ^2 is the variance of the model noises. Hence, the modification above for the estimation error is reflected in the $\mathbf{P}_{\mathbf{\theta},k}$ matrix in Eq. (9), i.e., $\mathbf{P}_{\mathbf{\theta},k} \to \mathbf{\Lambda}_{k+1}\mathbf{P}_{\mathbf{\theta},k}\mathbf{\Lambda}_{k+1}$, where $\mathbf{\Lambda}_{k+1}$ is a diagonal matrix with the jth diagonal element $\lambda_j(k+1)$. Consequently, the recursive solution for the variable parametric vector, $\hat{\mathbf{\theta}}_{k+1}$, is proposed to be obtained from Eqs.(7) – (11) as follows,

$$\hat{\mathbf{\theta}}_{k+1} = \hat{\mathbf{\theta}}_k + \mathbf{K}_{\mathbf{\theta},k+1}\left[\mathbf{y}_{k+1} - \mathbf{\varphi}_{k+1}\hat{\mathbf{\theta}}_k + \bar{\mathbf{\eta}}\hat{\bar{\mathbf{f}}}_{k+1|k+1}\right] \tag{18}$$

$$\hat{\bar{\mathbf{f}}}_{k+1|k+1} = -\mathbf{S}_{k+1}\bar{\mathbf{\eta}}^T[\mathbf{I} - \mathbf{\varphi}_{k+1}\mathbf{K}_{\mathbf{\theta},k+1}](\mathbf{y}_{k+1} - \mathbf{\varphi}_{k+1}\hat{\mathbf{\theta}}_k) \tag{19}$$

in which

$$\mathbf{K}_{\mathbf{\theta},k+1} = (\mathbf{\Lambda}_{k+1}\mathbf{P}_{\mathbf{\theta},k}\mathbf{\Lambda}_{k+1}^T)\mathbf{\varphi}_{k+1}^T\left[\mathbf{I} + \mathbf{\varphi}_{k+1}(\mathbf{\Lambda}_{k+1}\mathbf{P}_{\mathbf{\theta},k}\mathbf{\Lambda}_{k+1}^T)\mathbf{\varphi}_{k+1}^T\right]^{-1} \tag{20}$$

$$\mathbf{S}_{k+1} = \left[\bar{\mathbf{\eta}}^T(\mathbf{I} - \mathbf{\varphi}_{k+1}\mathbf{K}_{\mathbf{\theta},k+1})\bar{\mathbf{\eta}}\right]^{-1} \tag{21}$$

$$\mathbf{P}_{\mathbf{\theta},k} = (\mathbf{I} + \mathbf{K}_{\mathbf{\theta},k}\bar{\mathbf{\eta}}\mathbf{S}_k\bar{\mathbf{\eta}}^T\mathbf{\varphi}_k)(\mathbf{I} - \mathbf{K}_{\mathbf{\theta},k}\mathbf{\varphi}_k)(\mathbf{\Lambda}_k\mathbf{P}_{\mathbf{\theta},k-1}\mathbf{\Lambda}_k^T),\ k=1, 2, \ldots \tag{22}$$

In Eqs.(20) and (22), $\mathbf{\Lambda}_{k+1}$ is a diagonal matrix, referred to as the adaptive factor matrix, with diagonal elements $\lambda_1(k+1)$, $\lambda_2(k+1)$, $\ldots, \lambda_n(k+1)$, where $\lambda_j(k+1)$ is referred to as the adaptive factor for the estimated parameter $\theta_j(k+1)$ at $t_{k+1} = (k+1)\Delta t$. The determination of the adaptive factor matrix $\mathbf{\Lambda}_{k+1}$ has been described in Yang and Lin (2005). Also, the argument, $\mathbf{X}_{k+1}$, of $\mathbf{K}_{\mathbf{\theta},k+1}$ and $\mathbf{\varphi}_{k+1}$, which should be replaced by $\hat{\mathbf{X}}_{k+1|k+1}$ in Eq.(13), has been dropped in Eqs. (18)-(22) for simplicity of presentation.

IDENTIFICATION OF SUB-STRUCTURE

Consider a complex structure, such as the one shown in Fig.1(a), and suppose we are interested in monitoring some of the critical areas where damages may occur. For simplicity of presentation, let us consider only one critical area, consisting of 12 members as shown in Fig.1(a) by dashed lines, for the monitoring purpose. This critical area is referred to as the sub-structure as shown in Fig.1(b). From Fig.1(b), the sub-structure formed by these 12 critical members consists of 4 masses at nodes 6, 7, 17 and 18, referred to as the internal nodes, and 4 interface nodes at nodes 5, 8, 16 and 19. Let $\mathbf{u}_r(t)$ be the displacement vector of the internal nodes, and $\mathbf{u}_s(t)$ be the displacement vector of the interface nodes. Then, the equation of motion of the sub-structure can be expressed as

$$\begin{bmatrix}\mathbf{M}_{rs} & \mathbf{M}_{rr}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{u}}_s(t)\\ \ddot{\mathbf{u}}_r(t)\end{bmatrix}+\begin{bmatrix}\mathbf{C}_{rs} & \mathbf{C}_{rr}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{u}}_s(t)\\ \dot{\mathbf{u}}_r(t)\end{bmatrix}+\begin{bmatrix}\mathbf{K}_{rs} & \mathbf{K}_{rr}\end{bmatrix}\begin{bmatrix}\mathbf{u}_s(t)\\ \mathbf{u}_r(t)\end{bmatrix}=\begin{bmatrix}\mathbf{f}_r(t)\end{bmatrix} \tag{23}$$

in which $\mathbf{f}_r(t)$ is the external excitations to the sub-structure at the internal nodes, and the entire structure has been assumed to be linear elastic for simplicity of presentation.

The interaction effects at the interface nodes can be considered as the inputs (excitations) and the above equation can be expressed as

$$\mathbf{M}_{rr}\ddot{\mathbf{u}}_r(t)+\mathbf{C}_{rr}\dot{\mathbf{u}}_r(t)+\mathbf{K}_{rr}\mathbf{u}_r(t)=\mathbf{f}_r(t)-\mathbf{M}_{rs}\ddot{\mathbf{u}}_s(t)-\mathbf{C}_{rs}\dot{\mathbf{u}}_s(t)-\mathbf{K}_{rs}\mathbf{u}_s(t) \tag{24}$$

Now, some of the acceleration responses of the internal nodes may not be measured, referred to as the unknown outputs $\ddot{\mathbf{x}}^*$ in Eq.(1), and some of the accelerations at the interface nodes may not be measured, referred to as unknown inputs (excitations) $\mathbf{f}^*$ in Eq.(1). When some acceleration responses at the interface nodes are measured, their corresponding velocity and displacement responses are obtained by the Newmark- β integration method and these terms on the right hand side of Eq.(24) should be moved to the left hand side. Thus Eq.(24) can be cast appropriately into the form of Eq.(1), and the SNLSE-UI-UO solution presented previously can be used.

SIMULATION RESULTS

To demonstrate the accuracy of the sub-structure approach using SNLSE-UI-UO for parametric identifications and damage detections at critical locations, a long-span truss with the finite element formulation and an 8-story base-isolated building will be considered. For both examples, the sampling frequency is 500Hz for all measured responses.

Long-span Truss with Finite-Element Model

To demonstrate the accuracy of the substructure approach using SNLSE-UI-UO for parametric identifications and damage detections of substructures, the long-span truss in Fig.1(a) will be considered. This is a planar truss with 44 members (or elements) and a total of 41 DOFs [Bernal (2002), Gao and Spencer (2002)]. As observed from Fig.1(a), the system is statically indeterminate. Now, the substructure shown in Fig.1(b) will be identified and monitored. The finite-element substructure model consists of 12 members with uniform cross-section, 4 internal nodes, and 4 interface nodes, where each node has 2 DOFs (horizontal and vertical). Twelve critical members (or elements) to be monitored in Fig.1(b) are denoted as follows: member 1 (nodes 5-6), member 2 (nodes 6-7), member 3 (nodes 7-8), member 4 (nodes 16-17), member 5 (nodes 17-18), member 6 (nodes 18-19), member 7 (nodes 5-17), member 8 (nodes 6-18), member 9 (nodes 7-19), member 10 (nodes 6-17), member 11 (nodes 7-18), member 12 (nodes 6-16).

Let $\mathbf{M}_i$ and $\mathbf{K}_i$ be the local mass matrix and the local stiffness matrix, respectively, of the ith element (member) with an uniform cross-section in the local coordinate system,

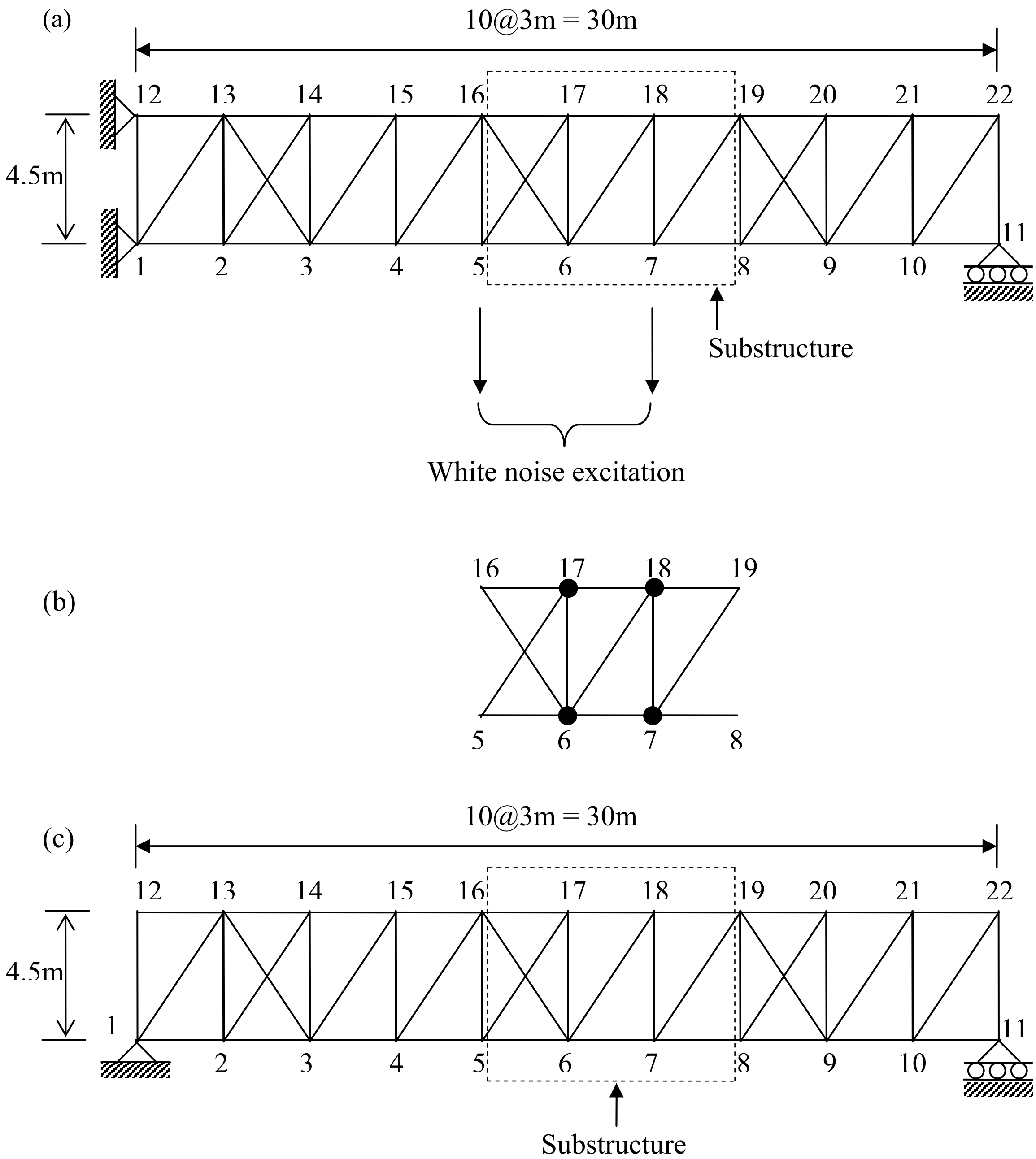

FIG 1. Long-span truss: (a) full structure with white noise excitation; (b) substructure; (c) full structure with earthquake excitation.

$$\mathbf{M}_i = \frac{\overline{m}_i L_i}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}; \quad \mathbf{K}_i = k_i \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

in which L_i and $\overline{m}_i$ are the length and the mass per unit length of the ith element (or member) of the sub-structure, respectively, and $k_i = E_i A_i / L_i$ is the equivalent stiffness parameter, where E_i and A_i are the Young's modulus and cross-sectional area of the ith element (or member), respectively. The local element mass and element stiffness matrices $\mathbf{M}_i$ and $\mathbf{K}_i$ are transformed into $\overline{\mathbf{M}}_i$ and $\overline{\mathbf{K}}_i$, which are the element matrices in the global coordinate system of the sub-structure, using the transformation matrix $\mathbf{T}$, i.e.,

$$\overline{\mathbf{M}}_i = \mathbf{T}^T \mathbf{M}_i \mathbf{T}; \quad \overline{\mathbf{K}}_i = \mathbf{T}^T \mathbf{K}_i \mathbf{T} \tag{26}$$

in which $\mathbf{T}$ is a (4×4) matrix with its (i, j) element, $\mathbf{T}_{ij}$, as: $\mathbf{T}_{11} = \mathbf{T}_{22} = \mathbf{T}_{33} = \mathbf{T}_{44} = \cos\theta$, $\mathbf{T}_{12} = \mathbf{T}_{34} = \sin\theta$, $\mathbf{T}_{21} = \mathbf{T}_{43} = -\sin\theta$ and $\mathbf{T}_{ij} = 0$ for other i and j, where θ = the angle between the local and global coordinates. Finally, the element mass and stiffness matrices $\overline{\mathbf{M}}_i$ and $\overline{\mathbf{K}}_i$ are expanded to (m×m) matrices denoted by $\widetilde{\mathbf{M}}_i$ and $\widetilde{\mathbf{K}}_i$, and the global mass and stiffness matrices $\mathbf{M}$ and $\mathbf{K}$ of the sub-structure, Fig.1(b), are obtained by summing up $\widetilde{\mathbf{M}}_i$ and $\widetilde{\mathbf{K}}_i$ for all the elements, i.e.

$$\mathbf{M} = \sum_{i=1}^{p} \widetilde{\mathbf{M}}_i; \quad \mathbf{K} = \sum_{i=1}^{p} \widetilde{\mathbf{K}}_i = \sum_{i=1}^{p} k_i \mathbf{S}_i \tag{27}$$

in which for simplicity of presentation $\widetilde{\mathbf{K}}_i$ is expressed in terms of $k_i \mathbf{S}_i$, where $k_i = E_i A_i / L_i$ is the equivalent stiffness parameter and $\mathbf{S}_i$ is a (m×m) matrix of the ith element. In Eq.(27), p is the total number of elements (members).

In the literature, the Rayleigh damping is usually assumed and the damping matrix $\mathbf{C}$ is expressed as:

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \tag{28}$$

in which α and β are the mass-proportional and the stiffness-proportional damping coefficients. All the truss members are made of steel (with E = 200 Gpa) with an area of 64.5 cm^2. For simplicity of computation, the element mass matrix $\mathbf{M}_i$ is approximated by a diagonal matrix with diagonal element 1.75×10^5 kg [Bernal (2002)], and hence the global mass matrix $\mathbf{M}$ is diagonal after transformation. The structural parameters are: $k_i = 430$ MN/m (i = 1, 2, …, 6), $k_i = 238.52$ MN/m (i = 7, 8, 9, 12), $k_i = 286.67$ MN/m (i = 10, 11), $\alpha = 0.1064\, s^{-1}$ and $\beta = 3.4 \times 10^{-3}$ s. With the structural properties above, the first three natural frequencies and the corresponding modal damping ratios are: ω_i = 0.64, 1.19 and 1.54 Hz, and ζ_i = 2%, 1.99% and 2.2%. Two different cases for the long-span truss subject to different external excitations will be considered in the following.

Case 1 (White Noise Excitation): Consider that the truss shown in Fig.1(a) is subject to two vertical white noise excitations applied vertically at nodes 5 and 7. The measured responses include: (i) the horizontal and vertical accelerations at all interface nodes and internal node 18, (ii) the horizontal acceleration of internal node 6, and (iii) the vertical accelerations of internal nodes 7 and 17. Note that the horizontal accelerations of internal nodes 7 and 17, the vertical acceleration of internal node 6, and the white noise excitation $f^*(t)$ at node 7 are not measured (unknown). All the measured quantities are simulated by superimposing the theoretically computed quantities with the corresponding stationary white noise with a 2% noise to signal ratio. The unknown quantities to be identified include: α, β, k_i (i =1, 2, …, 9), the state vector, and the unknown white noise excitation $f^*(t)$ at node 7.

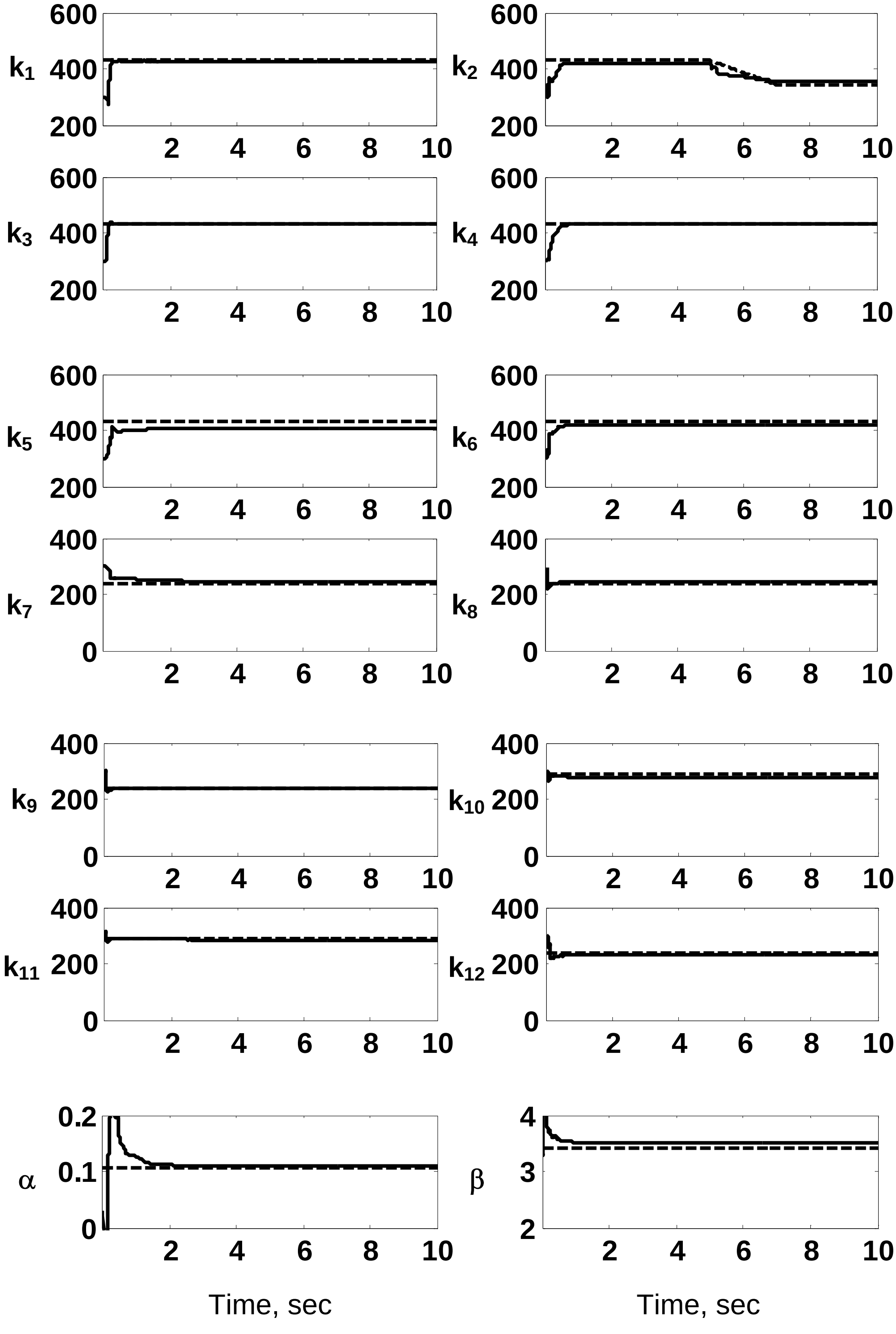

FIG 2. Identified parameters for a substructure of a long span truss (Case 1); (damage pattern 1) k_i in MN/m, α in s^{-1} and β in 10^{-3} sec.

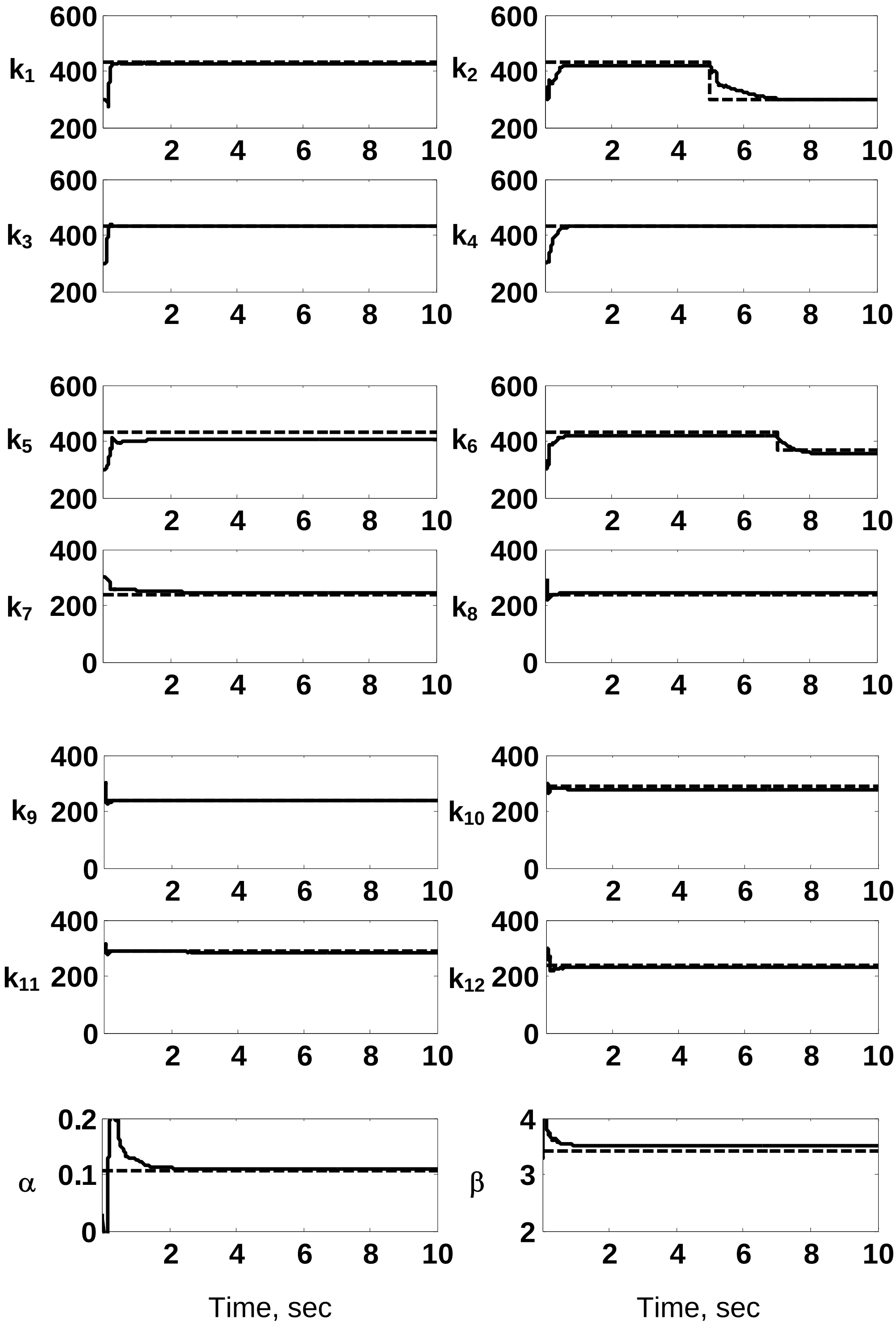

FIG 3. Identified parameters for a substructure of a long span truss (Case 1); (damage pattern 2) k_i in MN/m, α in s^{-1} and β in 10^{-3} sec.

The initial guesses for α, β and k_i are $\alpha_0 = 0.2\,s^{-1}$, $\beta_0 = 6\times10^{-3}\,s$, $k_{i,0} = 300\,MN/m$ (i = 1, 2, …, 12), respectively. The initial values for the state vector and the unknown excitation are zero, and the initial gain matrices $\mathbf{P}_{\boldsymbol{\theta},0}$ and $\overline{\mathbf{P}}_{0|0}$ are taken to be $\mathbf{P}_{\boldsymbol{\theta},0} = 10^{-2}\mathbf{I}_{25}$ and $\overline{\mathbf{P}}_{0|0} = 10^{6}\mathbf{I}_{16}$.

Two damage patters are considered. For damage pattern 1, a damage occurs at t = 5 sec, at which time the equivalent stiffness k_2 of member 2 in Fig.1(b) is reduced linearly from 430 MN/m to 344 MN/m (20% reduction) within 2 seconds. Based on the proposed adaptive SNLSE-UI-UO technique, the identified structural parameters are presented in Fig.2 as solid curves. Also shown in Fig.2 as dashed curves are the theoretical results for comparison.

For damage pattern 2, a damage occurs at t = 5 sec, at which time the equivalent stiffness k_2 of member 2 is reduced abruptly from 430 MN/m to 301 MN/m, then another damage occurs at t = 7 sec, at which time the equivalent stiffness k_6 of member 6 is reduced abruptly from 430 MN/m to 365.5 MN/m. Based on the proposed adaptive SNLSE-UI-UO technique, the identified structural parameters are presented in Fig.3 as solid curves, whereas the theoretical results are shown as dashed curves for comparison. The identified white noise excitation $f^*(t)$ at node 7 for a segment from 2 to 2.2 seconds is presented in Fig.4 as a solid curve, whereas the dashed curve in the same figure is the theoretical result. It is observed from Figs.2-4 that the proposed adaptive SNLSE-UI-UO technique tracks the substructural parameters, their variations due to damage, and the unknown excitation very well.

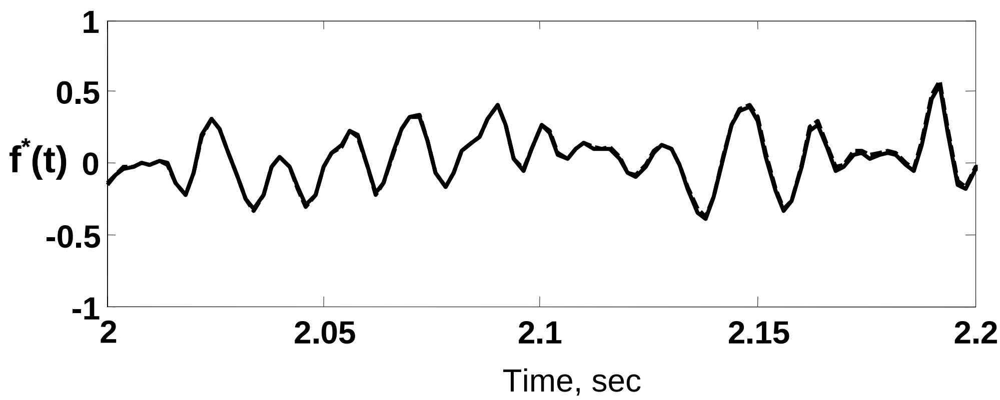

FIG 4. Identified unknown white noise excitation for long-span truss;
unit of $f^*(t)$ in 10^4 N.

Case 2 (Earthquake Excitation): Now, the left supports of the long-span truss in Fig.1(a) is modified so that the structure resembles the truss bridges as shown in Fig.1(c). Suppose this truss bridge is subject to the El Centro earthquake with a peak ground acceleration of 2g (PGA = 2g). The measured responses include: (i) the horizontal and vertical accelerations of all interface nodes and internal nodes 6 and 18, and (ii) the horizontal accelerations of internal nodes 7 and 17. Note that the vertical accelerations of internal nodes 7 and 17, and the earthquake ground acceleration $\ddot{x}_0(t)$ are not measured (unknown). All the measured quantities are simulated by superimposing the theoretically computed quantities with the corresponding stationary white noise with a 2% noise to signal ratio. In this case, the RMS of a particular response signal is computed from the temporal average over 30 seconds. The unknown quantities to be identified include: α, β, k_i (i =1, 2, …, 12), the state vector, and the unknown earthquake ground acceleration $\ddot{x}_0(t)$.

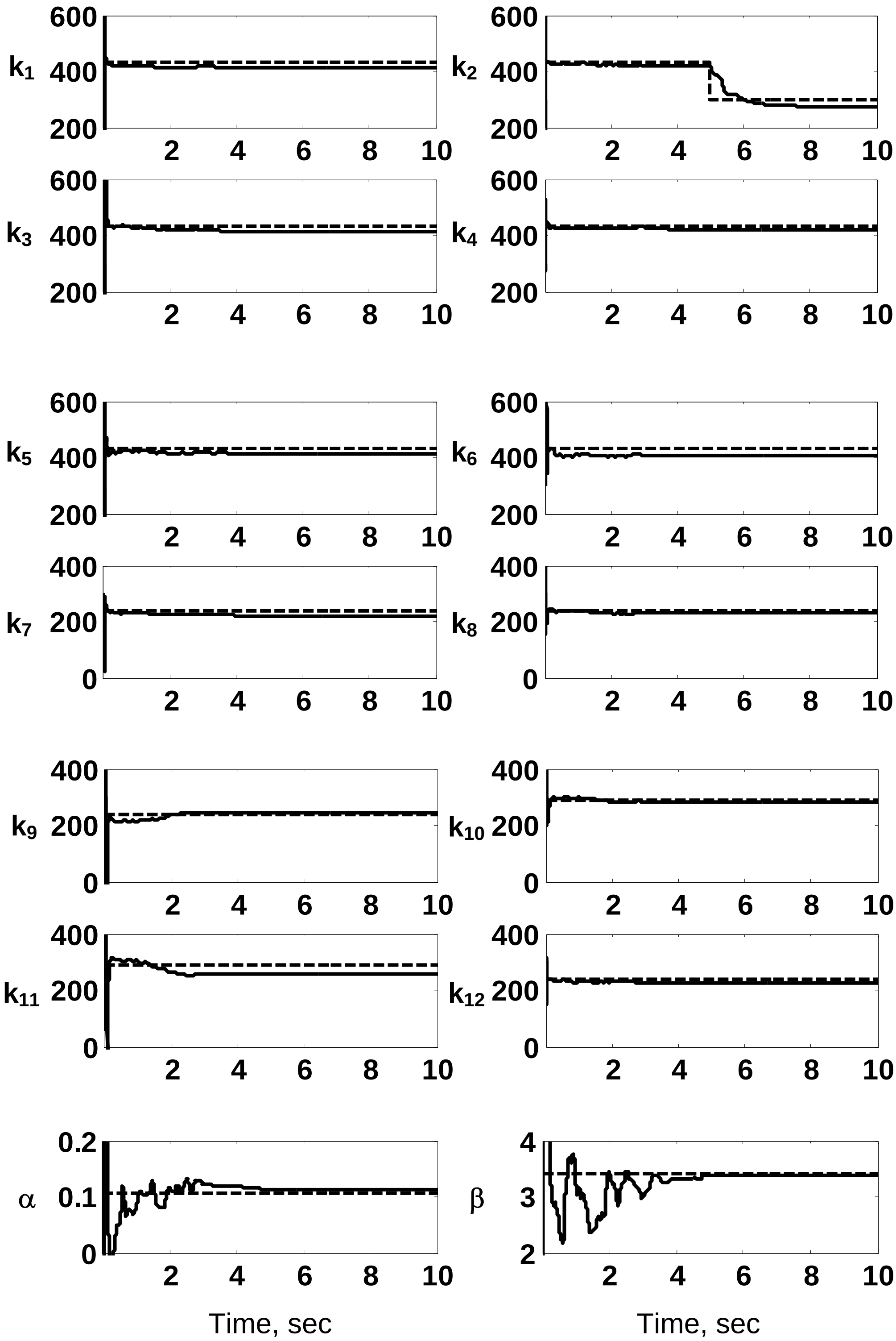

FIG 5. Identified parameters for a substructure of a long span truss (Case 2); (damage pattern 1) k_i in MN/m, α in s^{-1} and β in 10^{-3} sec.

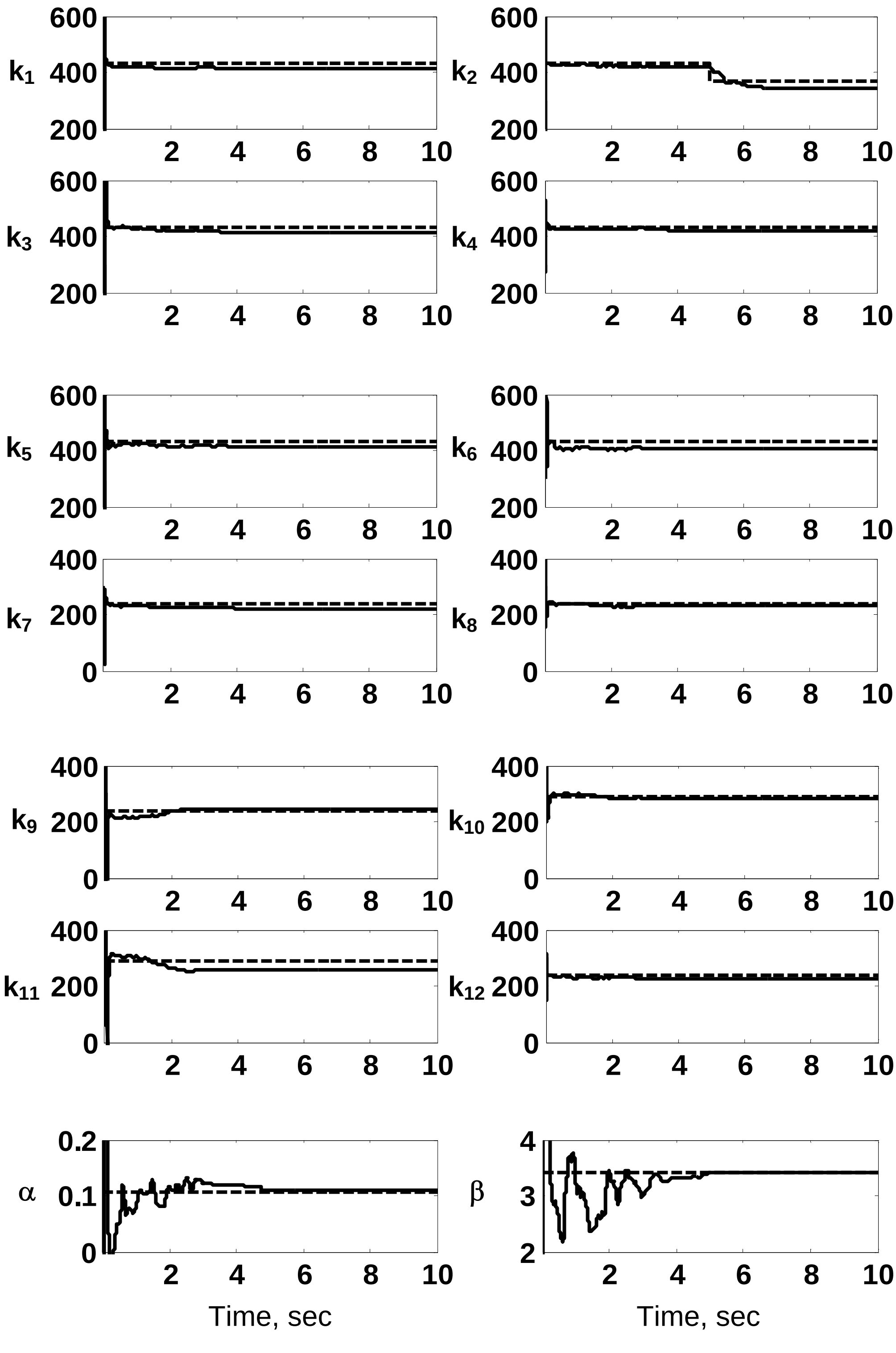

FIG 6. Identified parameters for a substructure of a long span truss (Case 2); (damage pattern 2) k_i in MN/m, α in s^{-1} and β in 10^{-3} sec.

Suppose a damage occurs at t = 5 sec, at which time the equivalent stiffness k_2 of member 2 in Fig.1(b) is reduced abruptly from 430 MN/m to 301 MN/m (30% reduction). The following assumed initial values and matrices are identical to that of Case 1: (i) initial state variables , (ii) initial unknown excitation, (iii) initial parametric values k_i (i = 1, 2, …, 12), α and β, and (iv) $\mathbf{P}_{\theta,0}$ and $\overline{\mathbf{P}}_{0|0}$. Based on the proposed adaptive SNLSE-UI-UO technique, the identified structural parameters are presented in Fig.5 as solid curves. Also shown in Fig.5 as dashed curves are the theoretical results for comparison.

The damage case above is severe. To show the capability of our proposed approach in detecting small damages, the same damage pattern above is considered except that the reductions of k_2 is smaller, where k_2 is reduced abruptly from 430 MN/m to 365.5 MN/m (15% reduction), at t = 5 sec. The identified structural parameters are presented in Fig.6 as solid curves, whereas the dashed curves are the theoretical results for comparison. Further, the identified earthquake ground acceleration $\ddot{x}_0(t)$ for a segment from 2 to 5 seconds is presented in Fig.7 as a solid curve, whereas the dashed curve in the same figure is the theoretical result. It is observed from Figs.5-7 that the proposed adaptive SNLSE-UI-UO technique tracks the substructural parameters, their variations due to damage, and the unknown excitation very well.

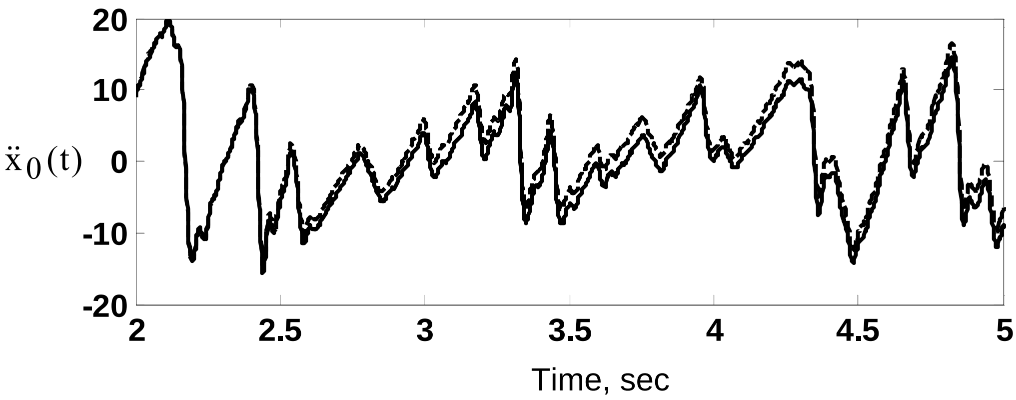

FIG 7. Identified unknown earthquake ground acceleration for long-span truss; unit of $\ddot{x}_0(t)$ in m^2/s.

8-story Base-Isolated Building

Consider an eight-story shear-beam type building subject to an earthquake ground acceleration $\ddot{x}_0(t)$, as shown in Fig.8(a). The properties of the building are as follows: (i) the mass of each floor is identical with m_i = 345.6 metric tons; (ii) the stiffness k_i (i = 1,2,…,8) of eight-story units are 340.4, 325.7, 284.9, 268.6, 243, 207.3, 168.7 and 136.6 MN/m, respectively; (iii) the linear viscous damping coefficients c_i (i = 1,2,…,8) for each story unit are 490, 467, 410, 386, 348, 298, 243 and 196 kN·sec/m, respectively. A lead-core rubber bearing isolation system is used to reduce the response of the building. The stiffness restoring force of the lead-core rubber-bearing is model by [Wen (1989)]

$$F_{sb} = \alpha_b k_b x_b + (1-\alpha_b) k_b D_{yb} v_b \tag{29}$$

in which the subscript b stands for the base-isolation system, x_b is the drift of the isolator, k_b is the stiffness, α_b is the ratio of the post yielding stiffness to the pre-yielding stiffness, D_{yb} is the yielding deformation, and v_b is the hysteretic component. The hysteretic component, v_b, is modeled by

$$\dot{v}_b = D_{yb}^{-1}[A_b\dot{x}_b - \beta_b\,|\dot{x}_b\,||\,v_b\,|^{n_b-1}\,v_b - \gamma_b\dot{x}_b\,|\,v_b\,|^{n_b}] = f_b(x_b, v_b) \tag{30}$$

in which A_b, β_b, n_b and γ_b are parameters characterizing the hysteresis loop. Properties of the base-isolation system are: m_b = 450 metric tons, k_b = 180.5 MN/m, linear viscous damping c_b = 26.17 kN.sec/m, α_b = 0.6, D_{yb} = 4 cm, A_b = 1.0, β_b = 0.5, n_b = 3 and γ_b = 0.5. For a small amplitude vibration (linear), the first natural frequency is ω_1 = 5.24 rad/sec. The El Centro earthquake $\ddot{x}_0(t)$ with a peak ground acceleration of 0.3g (PGA = 0.3g) is considered as the external excitation.

A substructure consists of the rubber bearing and the first story as shown in Fig.8(b) is considered for identification. In this example, the equation of motion is expressed in terms of the coordinate x_i representing the inter-story drift of the ith story. Two different cases will be considered.

Case 1: Earthquake excitation is measured. The absolute accelerations $\ddot{x}_b$ and $\ddot{x}_1$, and the El Centro earthquake ground acceleration $\ddot{x}_0(t)$ are measured. All measured quantities are simulated by superimposing the theoretically computed quantities with the corresponding stationary white noise with a 2% noise to signal ratio. In this case, the RMS of a particular response signal is computed from the temporal average over 30 seconds. Parameters α_b, D_{yb}, A_b and n_b are assumed to be known constants. The unknown quantities to be identified are: c_1, k_1, c_b, k_b β_b and γ_b, as well as the state vector of the substructure.

Suppose a damage occurs at t = 15 sec, at which time the equivalent stiffness k_b is reduced abruptly from 180.5 MN/m to 144.4 MN/m (20% reduction). The initial guesses for c_i, k_i, c_b, k_b, β_b and γ_b are: $c_{i,0}$ = 300 kN·sec/m, $k_{i,0}$ = 100 MN/m (i = 1, 2,…,8), $c_{b,0}$ = 10 kN·sec/m, $k_{b,0}$ = 10 MN/m , $\beta_{i,0}$ = 1, and $\gamma_{i,0}$ = 1, respectively. The initial values for the state variables are zero, and the initial gain matrices $\mathbf{P}_{\theta,0}$ and $\overline{\mathbf{P}}_{0|0}$ are taken to be $\mathbf{P}_{\theta,0} = 10^{10}\mathbf{I}_6$ and $\overline{\mathbf{P}}_{0|0} = \mathbf{I}_4$. Based on the proposed adaptive SNLSE-UI-UO technique, the identified parameters are presented in Fig.9 as solid curves. Also shown in Fig.9 as dashed curves are the theoretical results for comparison. It is observed from Fig.9 that the proposed approach is able to track the structural parameters and their variations due to damage.

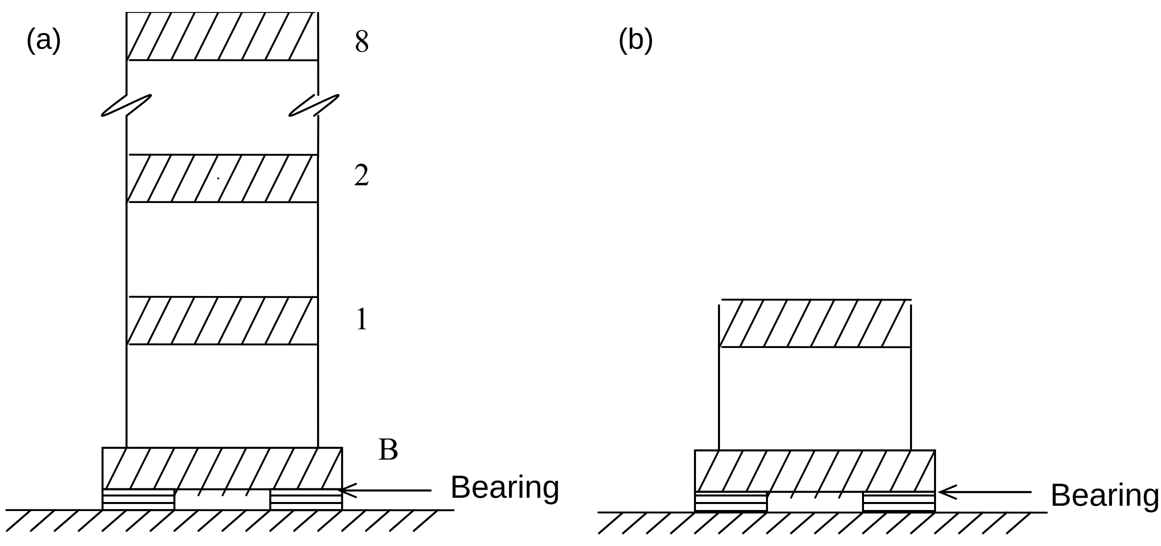

FIG 8. An 8-story base-isolated building: (a) full structure; (b) substructure.

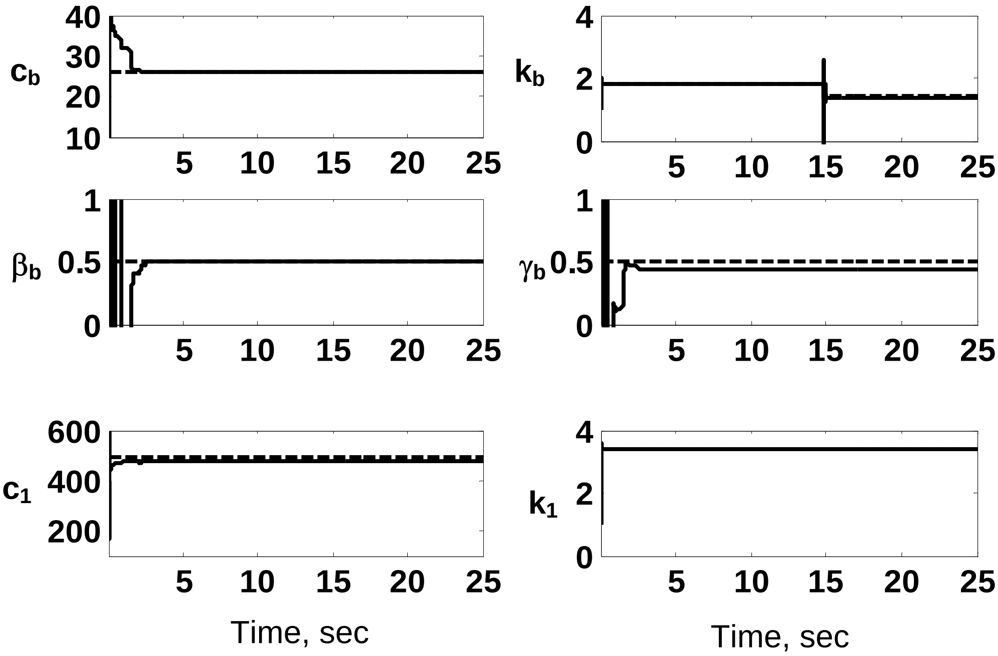

FIG 9. Identified parameters for a substructure of a 8-story base-isolated building (Case 1); k_b in 10^4 kN/m, k_1 in 10^5 kN/m, c_b and c_1 in kN.s/m.

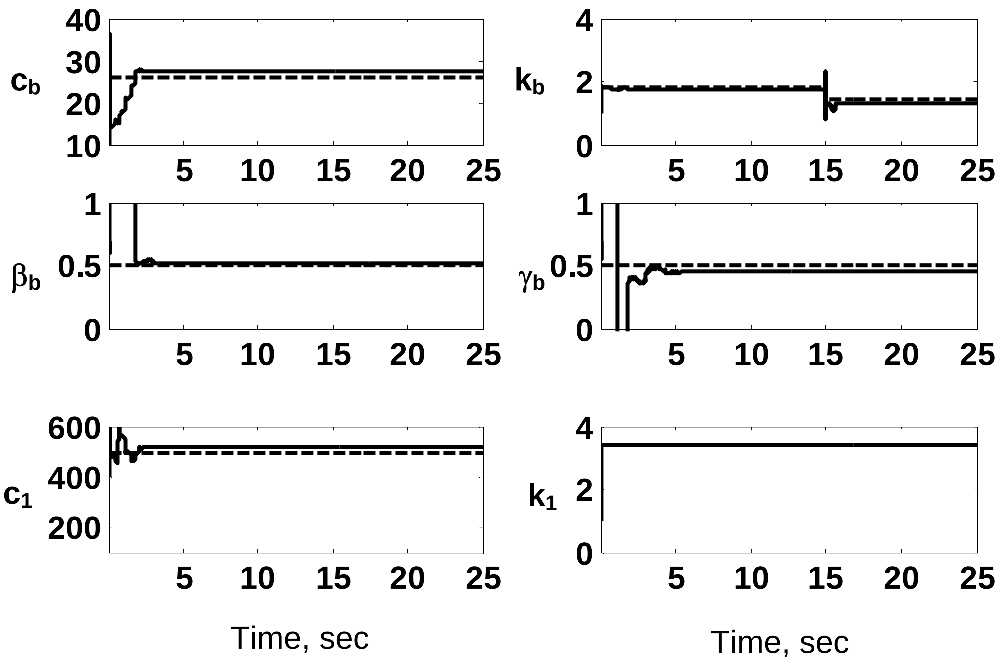

FIG 10. Identified parameters for a substructure of a 8-story base-isolated building (Case 2); k_b in 10^4 kN/m, k_1 in 10^5 kN/m, c_b and c_1 in kN.s/m.

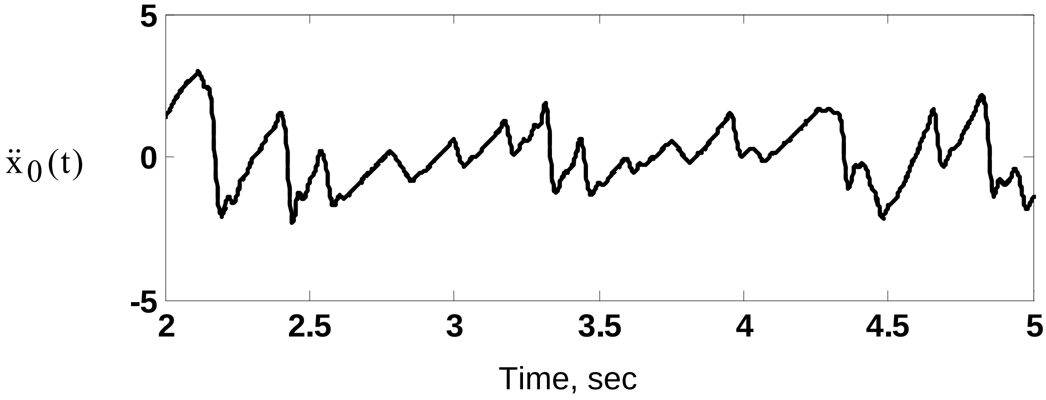

FIG 11. Identified unknown earthquake ground acceleration for 8-story base-isolated building; unit of earthquake acceleration $\ddot{x}_0(t)$ in m/s^2.

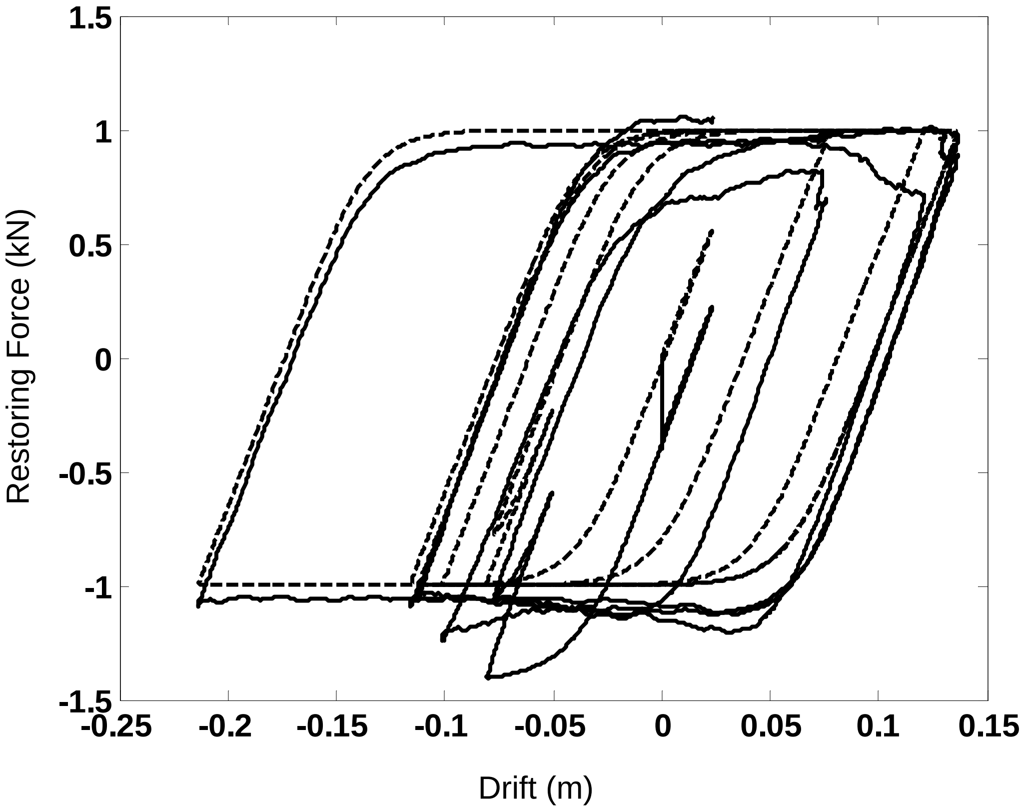

FIG 12. Identified hysteresis loops for the rubber-bearing of 8-story base-isolated building.

Case 2: Earthquake excitation is not measured. In this case, inter-story drifts x_b, x_1 and x_2 are measured. The earthquake ground acceleration $\ddot{x}_0(t)$ is not measured and hence it is unknown. The measured drifts were simulated by superimposing the theoretically computed quantities with the corresponding stationary white noise with a 2% noise to signal ratio. Finally, the inter-story accelerations $\ddot{x}_b$, $\ddot{x}_1$ and $\ddot{x}_2$ were computed by differentiations. Similar to Case 1, parameters α_b, D_{yb}, A_b and n_b are assumed to be known constants. The unknown parameters to be identified are: c_1, k_1, c_b, k_b β_b, γ_b, the unknown earthquake excitation $\ddot{x}_0(t)$, and the state vector of the substructure.

Suppose a damage occurs at t = 15 sec, at which time the equivalent stiffness k_b is reduced abruptly from 180.5 MN/m to 144.4 MN/m (20% reduction). The initial unknown excitation is zero, and the following assumed initial values and matrices are identical to that of Case 1 above: (i) initial state variables , (ii) initial parametric values c_1, k_1, c_b, k_b β_b and γ_b, and (iiii) $\mathbf{P}_{\boldsymbol{\theta},0}$ and $\overline{\mathbf{P}}_{0|0}$. Based on the proposed adaptive SNLSE-UI-UO technique, the identified parameters are presented in Fig.10 as solid curves. Also shown in Fig.10 as dashed curves are the theoretical results for comparison. The identified earthquake ground acceleration $\ddot{x}_0(t)$ for a segment from 2 to 5 seconds is presented in Fig.11 as a solid curve, whereas the dashed curve is the theoretical result. It is observed from Figs.10 and 11 that the proposed approach is able to track both the structural parameters and their variations due to damage, as well as the unknown earthquake excitation. Finally, the predicted hysteresis loops for the stiffness restoring force F_{sb} versus the drift x_b of the base isolator are presented in Fig.12 as solid curves, whereas the dotted curves represent the theoretical results. As shown in Fig.12, the predictive capability of the proposed approach is quite reasonable.

CONCLUSIONS

In this paper, the recently proposed adaptive sequential nonlinear least-square estimation with unknown inputs and unknown outputs (SNLSE-UI-UO) [Yang, et al (2006d)] along with the sub-structure approach have been used to identify structural damages at critical locations of a complex structure. This proposed approach allows for the damage monitoring of critical sub-structures without the need of information for the global complex structure, thus reducing significantly the total number of sensors required. Even for the critical sub-structure, the external excitations (inputs) and some acceleration responses (outputs) are not required to be measured, again reducing the required number of sensors. Simulation results using a long-span truss with finite-element formulation and an 8-story base-isolated hysteretic building demonstrate that the proposed approach is capable of identifying the changes of structural parameters, leading to the identification of structural damages.

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