NONLINEAR DYNAMIC RESPONSE OF STRUCTURE UNDERGOING PLASTIC DEFORMATION

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- The curve is nonlinear.
- What is the physical reason for it? The nonlinearity can be due to plastic deformation, fracture, or other causes.



A realistic material model

 We need a realistic material model in order to investigate the characteristics of data spectrum.

Material model (constitutive equations) for plastic deformation should be used in the analysis.

Plasticity: Difference between nonlinear elastic and plastic deformations can be observed during unloading.



- A36 steel, $\varepsilon_y = 0.0012$
- Change of 1.2 mm in a 1m bar, beyond that -> plastic range
- To measure strain: $\epsilon = \frac{\text{change in length}}{\text{original length}}$
- Measure the change in two marked lines on the specimen (caliper)
- Extensometer, 10^-4 accuracy
- Strain gauges,10^-6 accuracy
- Small strain $d\epsilon_{ij} = d\epsilon_{ij}^{e} + d\epsilon_{ij}^{p}$
- Small strain (infinitesimal) < 5% Large strain (finite strain) > 5%

Hysteresis, energy dissipation



Comparison between FFT and HHT

- Traditional data analysis (FFT): Linear and stationary
- HHT (Hilbert-Huang Transformation) adaptive analysis : Spectral analysis for nonlinear and nonstationary data.

• FFT:
$$x(t) = \kappa \sum_{j} a_{j} e^{i\omega_{j}t}$$

• HHT:
$$x(t) = \kappa \sum_{j} a_{j}(t) e^{i \int \omega_{j}(\tau) d\tau}$$

Special characteristics can be better identified:

- * Broadening of spectral peak serves as an indication that the load has over-strained the structure.
- * Frequency downshifting serves as an indication that the structure had softened or yielded.

Example: One story structure subjected to shear harmonic loading

$$F(t) = F_0 \sin \omega t$$





Bilinear ($F_0=130$ kN, $\omega=8\pi$ rad/s, ke=350,000 kN/m, kp=100,000 kN/m,

- The lower right figure shows Hilbert spectrum, plotting frequency vs time. It shows wavelet when plastic strain occurs.
- Integration of this figure wrt time gives the figure on the lower left, plotting energy vs frequency. Red line shows Hilbert spectrum, while green line shows Fourier spectrum.