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AN IMPROVED CAPACITY SPECTRUM METHOD BASED ON INELASTIC DEMAND SPECTRA

Xiao Mingkui[1], Dong Yinfeng[2], Liu Gang[3], and Cheng Guangjun[4]

ABSTRACT

The capacity spectrum method which intervenes between static inelastic analysis and elastic dynamic analysis is a simplified method to analysis the elastoplastic response and evaluate the performance of regular structures under strong ground motions to a certain extent of approximation. To address the existing problems, two aspects of improvements are made on the present capacity spectrum method. Firstly, by using the yield strength factor as an elastoplastic index, the inelastic demand spectra corresponding to the design acceleration response spectra specified in the Code for Seismic Design of Buildings in China are defined and formulated, while in the conventional methods the elastic demand spectra of high damping ratio are commonly used and can not reflect the structural elastoplastic response demand precisely. Secondly, according to the energy equivalent criterion the yield displacement and yield force of the equivalent SDOF system are determined based on the structural design parameters directly which are usually determined by bilinear modeling of Pushover curves of structures under area equivalent assumption in some present codes as FEMA-356 and Euro Code 8 specified. The comparison of the results shows the improved method is more simple and precise which can be easily carried out in structural seismic design.

Keywords: Capacity spectrum method, Inelastic demand spectra, Energy equivalent criteria

INTRODUCTION

The elastoplastic static analytic method is the primary simplified method for evaluating performance of seismic structures, which includes the pushover method and the capacity spectrum method. For the capacity spectrum method, the pushover method must be relied on to evaluate the performance of seismic structures and it can be viewed as an improvement on the pushover method. The development and improvement of both the simplified methods is keystone to carry out the performance based design in practice.

The capacity spectrum method (Freeman et al., 1975; Moehle, 1992; Fajfar, 1999) which intervenes between static inelastic analysis and elastic dynamic analysis is a simplified method to analysis the elastoplastic response and evaluate the performance of regular structures under strong ground motions to a certain extent of approximation. The method is mainly implemented by superposition of the capacity spectrum curve and the inelastic demand spectra curve to evaluate responses performance of structures.

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[1] Professor, College of Civil Engineering, Chongqing University, Chongqing, P. R. China, xmkxy@yahoo.com.cn
[2] Instructor, College of Civil Engineering, Chongqing University, Chongqing, P. R. China, yhvhson@cta.cq.cn
[3] Instructor, College of Civil Engineering, Chongqing University, Chongqing, P. R. China
[4] Instructor, College of Civil Engineering, Chongqing University, Chongqing, P. R. China

Two curves are needed in the capacity spectrum method. One is the inelastic demand spectrum curve which represents the demand for seismic capacity of structures under a given ground motion. Using the time-history analysis method, a given ground motion is input to a series of SDOF systems with the natural frequencies distributed in a certain range, then the corresponding displacement, velocity and acceleration responses can be found respectively. The demand spectrum curve, in which the abscissa and the ordinate are the displacement and acceleration responses separately, can be found by time-history analysis. Another curve is the capacity spectrum curve which represents the capacity of lateral drift of a structure. First, the top displacement and base shear force can be found by using the pushover analysis method in which some pattern of lateral forces are loaded to the structure gradually until it is destroyed. Then, the top displacement and base shear force is converted to the acceleration-displacement curve of the equivalent SDOF system called capacity spectrum curve. The performance and failure state of seismic structures can be evaluated qualitatively with the intersection point of the two curves.

In conventional capacity spectrum method (Freeman et al., 1975), the elastic demand spectrum is often adopted, which would lead to the demand for seismic capacity of a structure is overvalued. Therefore, a variety of improvement methods, e.g. elastic demand spectra with high damp ratio, are proposed by many researchers in the world (Miranda et al., 1994; Kowalsky et al, 1995; Calvi, 1999). The elastic demand spectra with high damp ratio means that the dissipation of energy yielded by inelastic deformation of seismic structures is equated by increscent damp ratio, so that the effects of elastoplastic response can be take into account. The disadvantage of this method is no corresponding relationship exists between the Hysteretic Energy and high damp ratio. Compared with the high damp ratio elastic spectrum, the simplified reduction coefficient spectra suggested by other researchers, which use ductility reduced coefficient R_m to modify the elastic spectra, such as displacement coefficient method in FEMA273， inelastic spectra with ductility reduced coefficient R_m by Fajfar et al. (1999), are more theoretically reasonable. But it is difficulty to find ductility coefficient of whole structure. Therefore, a question for discussion is how to find elastoplastic spectra in which different kinds of influencing factors may be covered.

The determination of dynamic coefficient of the equivalent SDOF system must be discussed while the top displacement and base shear, found by using pushover method, is converted to the acceleration-displacement curve of the equivalent SDOF system. There are various discussions about dynamic coefficien of the equivalent SDOF system (EC8, 2003; FEMA356, 2000). But the discussions are improper in determination of initial rigidity and yield displacement. There are some questions needed to discussion about the method for determination of initial rigidity and yield displacement in EC8 and FEMA356.

To address the existing problems, two aspects of improvements are made on the present capacity spectrum method in this paper. Firstly, by using the yield strength factor as an elastoplastic index, the elastoplastic demand spectra are derived and formulated based on the design acceleration response spectra specified in the Code for Seismic Design of Buildings in China. Secondly, according to the energy equivalent criterion the yield displacement and yield force of the equivalent SDOF system are determined based on the structural design parameters directly which are usually determined by bilinear modeling of pushover curves of structures under area equivalent assumption in some present standards as FEMA356 and EC8 specified.

ELASTOPLASTIC DEMAND SPECTRA

To overcome the disadvantages of using elastic demand spectra with high damp ratio and reduction coefficient , by using the yield strength factor as an elastoplastic index, the elastoplastic demand spectra are derived and formulated based on the design acceleration response spectra specified in the Code for Seismic Design of Buildings in P.R.China (2001). The SDOF systems with natural period of vibration ranged 0.1-6.0s and earthquake waves with predomination period ranged 0.1-1.2s are selected for computing elastoplastic demand spectra by time-history method. The elastoplastic acceleration and displacement are founded and elastoplastic demand spectra are shown in Fig. 1. Limit to space of the paper, only two figures are shown.

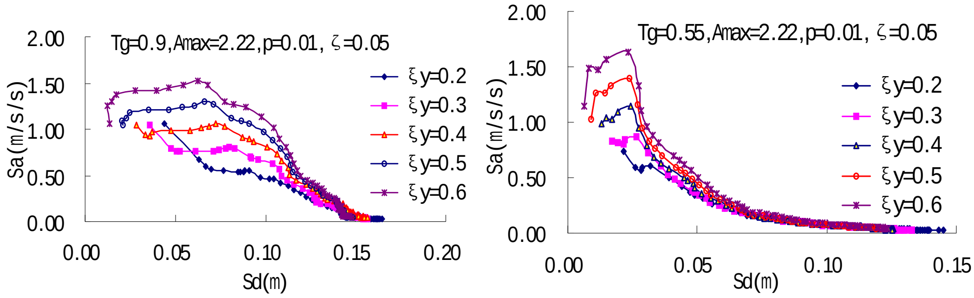

Figure 1. Elastoplastic Demand Spectrum

THE DISCUSS ABOUT EQUIVALENT YIELD DISPLACEMENT IN BILINEAR MODELING OF PUSHOVER CURVES

The base shear and top displacement found by using pushover method can be converted to bilinear modeling capacity spectrum curve and equivalent yield displacement d_y^* can be found in bilinear modeling capacity spectrum curve in EC8 (2003). Firstly, the top displacement d and base shear F_b can be found by using pushover method which loaded with stated distributing along the height of structures loads. The coefficients of the equivalent SDOF system are given by

$$F^* = \frac{F_b}{\Gamma}, \qquad d^* = \frac{d}{\Gamma}, \qquad \Gamma = \sum_{j=1}^{n} m_j f_j \Big/ \sum_{j=1}^{n} m_j f_j^2,$$

where Γ is the coefficient concerned with vibration modes, usually only the first order coefficient is selected. $F^* - d^*$ curve as shown Fig. 2. F_y^* is ordinate value corresponding to maximum value d_m^* in the $F^* - d^*$ curve. The curve is simplified to idealized bilinear curve in which horizontal line is made through F_y^* and intersected with line which gradient is k_e. The abscissa magnitude of point of intersection is d_y^*. The determination of inclination k_e and d_y^* should be meet the condition that the area enclosed by bilinear curve and abscissa should equal to the area enclosed by $F^* - d^*$ curve and abscissa. $d_m^* = d_t^* / 1.5$, moreover

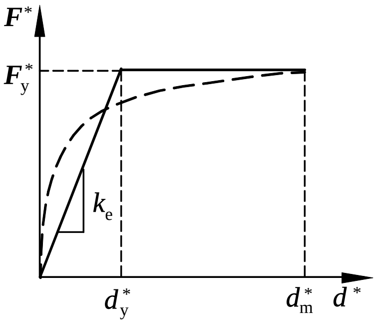

Figure 2. Idealization of Pushover curve in EC8

$$d_t^* = S_a(T^2/4p^2) \qquad T_e \geqslant T_c$$

$$d_t^* = S_a \frac{T^2}{4p^2}\frac{1}{q}\left[1+(q-1)\frac{T_c}{T}\right] \qquad T_e \geqslant T_c$$

d_t^* is target displacement while loading in pushover analysis, where

$$q = S_a/(F_y^*/m^*), \qquad m^* = \sum_{j=1}^{n} m_j f_j .$$

By same pushover method, the base shear and top displacement and equivalent yield displacement d_y^* in the bilinear capacity spectrum curve are found in FEMA356. But there are differences only in deciding d_y^* of bilinear curve. It is suggested by FEMA356 that the initial stiffness estimate is governed by the requirement that the actual and idealized curves intersect at $0.6F_y^*$, and post-yield stiffness is governed by the requirement for the curves to meet at the target displacement. F_y and d_y are selected by trial and error so as to give approximately equal areas under bilinear curve and $F^* - d^*$ curve in the target displacement, as shown in Fig. 3.

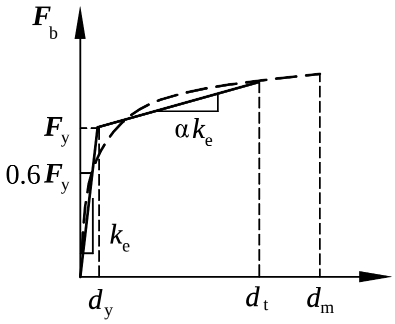

Figure 3. Idealization of Pushover curve in Fema356

The target displacement under design seismic wave is selected by the following formulae:

$$d_t = \Gamma S_a (T^2/4p^2) \qquad T_e \ge T_c$$

$$d_t = \Gamma S_a \frac{T^2}{4p^2}\frac{1}{q}\left[1+(q-1)\frac{T_c}{T}\right] \qquad T_e < T_c,$$

where T_e is idealized elastic periods, S_a is acceleration response spectra corresponding to T_e, T_c is predomination periods of ground movement. F_y^* and d_y^* can be found if F_y and d_y are multiplied by vibration mode coefficient Γ respectively.

The disadvantages of the methods proposed by EC8 and FEMA356 are computing trial and error is needed, and because there are difference of initial stiffness selected by using EC8 method and FEMA356 method respectively , the value of T^* gotten by EC8 method is greater than that gotten by FEMA356. If pushover curve is trilinear type, the difficult problem of computing trial and error occur to both EC8 and FEMA356 method.

It is suggested that the nonlinear characteristic of seismic structures are denoted by the floor strength coefficient x_Y in the paper. x_Y is expressed as following

$$x_Y(i) = h_p \frac{\Delta u_y(i)}{\Delta u_p(i)} = \frac{\Delta u_y(i)}{\Delta u_e(i)} = \frac{V_y(i)}{V_e(i)}, \qquad (1)$$

where h_p is an increscent coefficient of floor elastoplastic displacement. The value is from 1.3 to 2.2 for multi-story frames. $\Delta u_e(i),\ V_e(i)$ are floor elastic displacement and elastic shear which are found only by elastic analysis or by elastic response spectra vibration mode decomposition method. By vibration mode decomposition method in which elastic response spectra correspond to the Code for Seismic Design of Buildings in China are employed , the value of $\Delta u_e(i),\ V_e(i)$ in statistical sense can be found. The floor yield displacement and yield shear $V_y(i)$ can be found directly by the design cross section and design reinforcement number of the structure members. Then the equivalent yield shear are found according to first order vibration mode

$$F_y^* = \frac{\sum F_{yi} f_i}{\Gamma_1}. \qquad (2)$$

The equivalent elastic base shear F_e^* is found by base shear method. Then, the yield strength coefficient of equivalent SDOF system is

$$x_y^* = F_y^* / F_e^* = d_y^* / d_e^*. \qquad (3)$$

For considering the affect of all kinds of earthquake waves, several different kinds of earthquake waves are chosen and time-history method is employed to find the elastic maximum displacement of frames. Then the elastic displacement of equivalent SDOF system is found according to first vibration mode, and Eq. 3 is employed to find d_y^* and x_y^*. It is observed that the elastoplastic analysis do not needed if the method proposed by the paper is used. Even if the effects of all kinds of earthquake waves are required to consider, only elastic time-history analysis method is needed.

The rationality of the method is validated by identification method based the energy equivalent criterion. Yield displacement of equivalent elastoplastic SDOF system are identified by the criteria that the hysteretic energy per unit mass of original frame equal to that of the equivalent SDOF system. The formula which are created by Akiyama (1985 and 1988) and time-history method are employed to compute the hysteretic energy. The validate result is shown latterly by an example.

THE IMPROVED CAPACITY SPECTRUM METHOD BASED INELASTIC DEMAND SPECTRA AND ANALYTIC PROCESS

The improved capacity spectrum method, in which the inelastic demand spectra are derived from the design acceleration response spectra specified in the Code for Seismic Design of Buildings in China and the yield displacement and yield force of the equivalent SDOF system are determined based on the structural design parameters directly according to the energy equivalent criterion, is proposed in this contribution. The analytic process of improved capacity spectrum method is basically same as the analytic process of traditional capacity spectrum method. The different process are the elastoplastic demand spectrum should be chosen according to the characteristic period T_g of a given site firstly, then the pushover curve should be idealized to bilinear capacity spectrum curve by finding x_y^* of the equivalent SDOF system using the method proposed in the paper. The subsequent process is to superpose bilinear capacity spectrum curve and demand spectrum curve, find the intersection of two curves and displacement demand magnitude by the intersection point. Then convert displacement demand magnitude into top displacement of frame structure given by

$$d_{target} = \Gamma_1 f_{N1} D_t. \qquad (4)$$

The pushover analysis should be carried out again based on the target displacement d_{target} . The deformation of every member of seismic frame structure under target displacement d_{target} can be found by loading to the seismic frame structure gradually until the top displacement of seismic frame structure is to target displacement d_{target} . The performance of seismic frame structure is evaluated by the deformation of every member of seismic frame structure under target displacement d_{target} .

EXAMPLE ANALYSIS

To compare the differences between the improved capacity spectrum method proposed in the paper and the methods in EC8 and FEMA356, the x_y^* value of the equivalent SDOF system of a 5-floor, 3-bay frame is calculated under the earthquake wave for $T_g = 0.4$ with the capacity spectrum method and time-history method respectively. The results of the equivalent parameter x_y^* are shown in Table 1.

Table 1. The value of x_y^* calculated by identifying method of equivalent energy

Earthquake Waves	USA01385	USA00631	USA01083	USA01945	USA02138	CHI00056
x_y^*	0.36	0.36	0.30	0.64	0.45	0.37

The results of the equivalent mass value, period and yield displacement parameters of the frame are shown in Table 2.

Table 2. Coefficients of equivalent SDOF

	Results obtained by the proposed method (d$_e$ found by base shear method)	Results obtained by the method in FEMA356	Results obtained by the method in EC8(2003)
T^*(s)	1.114	1.099	1.311
D_y^*(m)	0.032	0.027	0.040
D_t^*(m)	0.077	0.076	0.074
F_y^*(kN)	301.095	264.448	275.513
x_y^*	0.415	0.360	0.548
M^*(kG)	296835.400		

Table 2 shows that the periods of the equivalent SDOF system is close to the natural period of the frame 1.06s, and x_y^* is about 0.4. It is observed from Table 2 that the results obtained by the method of the paper are closer to the results obtained by the method in the FEMA356. Considering the effects of earthquake waves, several different kinds of earthquake waves are chosen and time-history method is applied to find the elastic displacement d$_e$ and x_y^* of frame. The results are shown in Table 3.

The results show that x_y^* varies with different earthquake waves, while the mean value 0.491 has a little difference with the result computed by the method in FEMA356 and EC8. If more earthquake waves are chosen as input data when time-history method is carried out, more accurate estimate of x_y^* in statistical sense can be found. Therefore, it is suggested that the method can be used in computing x_y^* of the equivalent SDOF.

The overlay of the capacity spectrum curve, which is made by converting the base shear and top displacement of the frame computed by pushover method with inverted triangle loading pattern, and elastoplastic demand spectra curve, is shown in Fig. 4. The abscissa value of the intersection of the two curves is 0.061, while $x_y^* = 0.4$, can be found in Fig.4. The first mode coefficient of the frame Γ_1 is 1.24, by substituting $\Gamma_1 = 1.24$ to Eq. 4, the target displacement or top displacement of the frame is

$$d_{target} = d_{top} = 0.074\,\text{m}.$$

The Pushover analysis is carried out again based on the target displacement d_{target}. The deformations of all members of the frame structure under target displacement d_{target} are found by loading gradually until the top displacement is the target displacement d_{target}.

Table 3. x_y^* of equivalent SDOF

Wave name	Results obtained by the proposed method	Results obtained by the method in FEMA356	Results obtained by the method in EC8
USA01385 (L type)	0.434	0.387	0.461
CHI00056 (L type)	0.506	0.413	0.585
USA00631(M type)	0.527	0.465	0.440
USA01083(M type)	0.500	0.427	0.799
USA02138 (S type)	0.507	0.434	0.631
USA01945(S type)	0.470	0.407	0.464
Mean-Std.	0.457	0.396	0.425
Mean	0.491	0.422	0.563
Mean+Std.	0.523	0.449	0.702

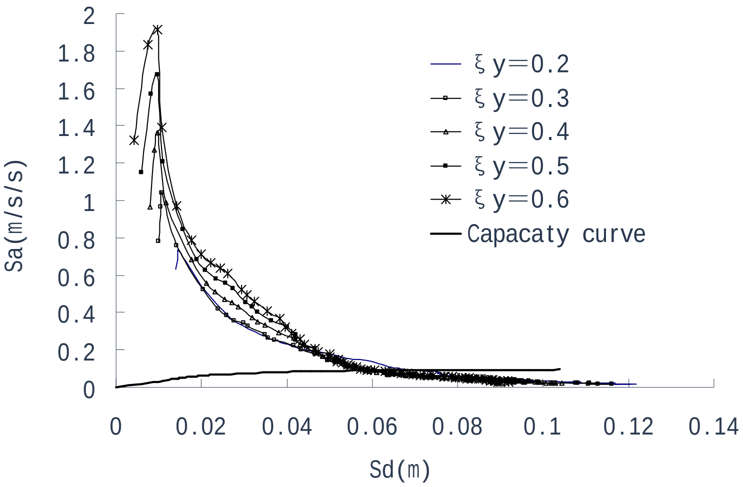

Figure 4. Capacity VS Requirement spectrum of 5-story frame

CONCLUSIONS

Two aspects of improvements are made on the present capacity spectrum method in the paper. The inelastic demand spectra by using the yield strength factor as an elastoplastic index and corresponding to the design acceleration response spectra specified in the Code for Seismic Design of Buildings in China are defined and formulated. According to the energy equivalent criterion, the equivalent SDOF system are determined based on the structural design parameters directly which are usually determined by bilinear modeling of pushover curves of structures under area equivalent assumption in some present codes as FEMA356 and EC8 specified. The comparison of results shows the improved method is more simple and precise which can be easily carried out in structural seismic design.

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