RELIABILITY ASSESSMENT OF SELF-CENTERING STEEL MOMENT FRAME SYSTEMS

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ABSTRACT

Researchers from various institutions have recently developed innovative steel moment frame systems for earthquake-resistant design that, following a major earthquake, have the potential to reduce or eliminate structural damage and return to its original vertical position (i.e. self-center). As the development of these steel moment frames with self-centering behavior progresses, it will be necessary to specify design parameters and a workable design method. This will require a good understanding of the sensitivity of the structure to changes in these parameters as well as the reliability of structures built based on the proposed method. This research investigates a risk assessment methodology (using Monte Carlo simulation) of the seismic response of self-centering steel moment frame prototype buildings. First, we perform a series of conditional seismic reliability assessments of the structure. Synthetically generated earthquakes with magnitudes and distances within narrow ranges that are consistent with ground-motion attenuation curves for a specified return period a specific building site are generated and applied to the structure, and peak relative rotations between the beam and column are recorded. This data is then used to generate demand curves (with respect to relative rotation) for the structure. From these, one can assess the risk of a particular design at a specific site having significant structural damage. Next, a study is performed to determine the capacity of gap relative rotation in relation to limit state attainment. The results of both studies are then combined to determine the overall reliability of the structure which be used to develop a reliability-based seismic design procedure for these self-centering steel moment frames.

Keywords: self-centering, moment frames, reliability, performance-based design

INTRODUCTION

Innovative self-centering steel moment resisting frame (SC-MRF) systems for earthquake-resistant design have recently been developed. Following a major earthquake, this system has the potential to reduce or eliminate structural damage and return to its original vertical position (i.e. self-center). In a SC-MRF, post-tensioned strands (or bars) run parallel to the beam and compress the beam against the column face. When the moment in the connection overcomes the moment resisted by the post-tensioned strands, a

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relative rotation develops between the beam and column ($\theta_r$). More details on this system can be found in Ricles et al. (2001); Garlock et al. (2005); Rojas et al. (2005); Christopoulos et al. (2002a).

This paper investigates a reliability assessment procedure for the seismic response of SC-MRFs. This methodology involves calculating the seismic demands on the SC-MRF and the structural capacity related to a specific limit state. In this paper, the limit state of strand yielding is used as a measure of capacity to illustrate the reliability assessment procedure. Other limit states can be used in manners similar to that shown here.

$\theta_r$, as shown in Figure 1, is selected as the measure of demand for this study since in a SC-MRF, $\theta_r$ can be related to many limit states such as strand yielding, beam buckling and energy dissipation device limit states. Two methods are proposed for developing the seismic demands on the prototype structure: a site characteristic method and a scenario method where magnitudes and distances of earthquakes are limited to a narrow ranges. Both methods use Monte Carlo simulations with the distance to and the magnitude of the seismic event as random variables. The prototype SC-MRF is subjected to a large number of earthquakes using randomly generated ground motions consistent with the site or scenario. At the end of each non-linear analysis, the peak $\theta_r$ is recorded.

| Figure 1: SC-MRF Connection Detail. |

Using the probabilistic demand and capacity curves generated through the simulations, the combined probability of the demand exceeding the capacity is calculated for every floor of the prototype frame. Such a method can be used to develop a reliability performance-based design approach for SC-MRF systems.

**STRUCTURAL MODEL**

The two primary methods of modeling SC-MRF systems involve the use of explicit gap elements, and the use, at each connection, of rotational springs that model the connection moment-rotation behavior. An analysis was performed by the authors, to determine the advantages and disadvantages of each method (Dobossy et al., 2006). It was found that while the rotational spring model has some limitations, owing to its computational efficiency, the rotational spring model is the best method for a study involving thousands of simulations such as the one at hand. Thus, the rotational spring model was chosen for this study.

The rotational spring model is a simplified approach to modeling SC-MRF systems where a rotational spring with a moment-rotation behavior representative of the behavior seen at a SC-MRF connection is inserted between the beam and column (Christopoulos et al., 2002a; Tsai et al., 2005; Priestley et al., 1999). A schematic of a typical inner column connection model is shown in Figure 2. In order to properly simulate the depth of the columns, slaved nodes are used to push the connection out from the
centerline of the column. It should be noted that this model does not take panel zone deformation into consideration. The rotational spring is given the moment-relative rotation ($\theta_r$) behavior shown in the Figure 2 inset. Dobossy et al. (2006) describes in detail how to determine the $M - \theta_r$ characteristics.

For the case study to demonstrate the methods proposed in this paper, the 6-story prototype frame “A” as described in Garlock et al. (2007a) and Garlock et al. (2007b) was used. Figure 3 shows an elevation of the frame. It consists of four 30-foot wide bays and six stories. The first story is 15-foot tall, while the subsequent stories are all 13-foot tall. To simulate the interior mass of the structure, and the method by which inertial forces are transferred to the frame during a seismic event, a lean-on column was used. At each floor of the lean-on column, an appropriate mass corresponding to the mass not directly supported by the exterior SC-MRF frame, is applied. The floor diaphragm flexibility is modeled with zero-length elements as described by Dobossy et al. (2006).

![Diagram](Figure 2: Model Rotational Spring schematic.)

![Diagram](Figure 3: Prototype Structure Elevation.)

**SEISMIC DEMAND**

The seismic demand variable of a structure is the value of a structural response such as maximum roof drift or interstory drift, under a given seismic condition (e.g., 100-year return period spectral acceleration). Typically this is calculated as a specific value (e.g., 5-cm or 10% maximum roof drift), but in reality is a site-specific distribution of values. In the case of a SC-MRF, the seismic demand variable of interest is relative rotation on each floor ($\theta_r$). $\theta_r$ is directly related to limit states of a SC-MRF such as post-tensioning strand yielding, energy dissipation device limit states and beam local buckling. This section outlines a method developed to generate demand curves for two different conditions: a specific site with known seismic sources and levels of activity (Site Characteristic Method), and for a specific “scenario” or event (Scenario Method), such as a 7.0 magnitude earthquake occurring at an epicentral distance of 20-30 kilometers.

For this research, a method based on Monte Carlo simulation of realistic synthetically generated earthquakes was developed. The general procedure for the two iterative methods is similar: the difference lies in how ground motions are selected. The first step in an iterative analysis is choosing parameters for the ground motion based on three factors: site soil conditions, distance to the event ($R$), and magnitude of the event ($M$). The selection of earthquake parameters is important, and varies greatly depending on the site and analysis type; more details are provided in the following two sections.

For both methods, after the parameters are chosen, acceleration time histories at the site location must be generated. Although any of several artificial accelogram methods may be used, for this research a method proposed by Sabetta and Pugliese (1996) was adopted due to the simplicity of the input requirements ($R$, $M$).
and soil depth) and the realistic nonstationary accelograms it generates. However, the attenuation relationships developed by Sabetta and Pugliese (1996) were based on limited seismic data (magnitude 4.4 to 6.8) from Italian quakes. To be applicable to a hypothetical site located in southern California, the attenuation (mainly of Arias Intensity) was updated using the empirical relationship developed by Travasarou et al. (2003).

Each time history of the acceleration is then applied at the base of the structure. In this research, the OpenSees software was used for the time-history analysis of the structural response (McKenna & Fenves, 2006). At each connection, the envelope of maximum and minimum $\theta_r$ is recorded for each ground motion. Many simulated ground motions for the site and its seismic setting are generated. After a large number of simulations are performed, a distribution is fitted to the data at each connection, resulting in a demand curve for the connection relative rotation.

Site Characteristic Method

To demonstrate the site characteristic method, a fictitious site with specified nearby (within 100km) seismic sources was created. Figure 4 shows a seismic source map of the area surrounding the site. The “x” indicates the site location, while the lines (1,2,3,4) refer to seismic sources. Each of the lines is then discretized into a number of point sources.

![Figure 4: Fictitious site source map.](image)

To generate earthquakes of correct magnitude and recurrence, Gutenberg-Richter law is used:

$$\ln(N(M)) = a - bM$$

where $N(M)$ is the number of earthquakes with magnitude exceeding $M$ in a given period (i.e., a year), $a$ is a recurrence rate constant for a given source, and $b$ is a constant which is typically taken as 1. The magnitude for the source is bound on the interval $M_{min} \leq M \leq M_{max}$ where $M_{min}$ is the minimum magnitude of interest, and $M_{max}$ is the maximum possible magnitude the source can generate. It should be noted that any seismic recurrence model could be used here, such as a modified Gutenberg-Richter model (Sornette & Sornette, 1999; Lombardi, 2003) or a mutually exclusive seismic-hazard source model (Field et al., 1999). Each source is given a recurrence constant, $a$ that corresponds to the source’s activity rate, and a maximum magnitude, $M_{max}$. From this, a recurrence curve is produced as shown in Figure 5.

The next step is to determine the minimum magnitude $M_{min}$, corresponding to each source’s contribution to the site seismic risk for a given mean return period. For this case study, the hundred-year mean return period was chosen. Assuming that the events associated with different seismic sources are statistically independent, the probability $P$ of a site-event occurring is:

$$P = 1 - \prod_{i=1}^{n}(1 - p_i)$$
where $n$ is the number of sources and $p_i$ is the probability of an event occurring at source $i$. For small values of $p_i$, this can be simplified to:

$$P \approx \sum_{i=1}^{n} p_i$$  \hspace{1cm} (3)

When looking at a specific seismic return period, some sources will contribute more than others. In order to determine the contribution of seismic sources, the measure of Arias Intensity ($I_A$) was used. A general form of Arias Intensity in terms of magnitude and distance ($R_i$) is:

$$I_A = \alpha_1 e^{\alpha_2 M} R_i^{-\alpha_3}$$  \hspace{1cm} (4)

where $\alpha_1$, $\alpha_2$, and $\alpha_3$ are site specific constants. Solving Eq. 4 for $M$, then substituting into the Gutenberg-Richter equation (Eq. 1) and simplifying yields:

$$P[M \geq M(I_A)] = \begin{cases} C \frac{a_i}{R_i^{\alpha_3/\alpha_2}}, & M \leq M_{i,\text{max}} \\ 0, & M > M_{i,\text{max}} \end{cases}$$  \hspace{1cm} (5)

where

$$C = \frac{a_{i1}^{1/\alpha_2}}{I_A^{1/\alpha_2}}$$  \hspace{1cm} (6)

Finally, by assuming the approximation for $P$ in Eq. 3, one can solve for the source recurrence rate contribution to the overall site probability, $p_i$:

$$p_i = \frac{P}{\sum_{j=1}^{n} \frac{a_j}{R_j^{\alpha_3/\alpha_2}}}$$  \hspace{1cm} (7)

In cases where the magnitude necessary for a given $p_i$ is greater than the maximum magnitude producible by the source, the source no longer contributes to the overall recurrence at the site, and $p_i$ is zero.

From this, an upper bound of $p_i$ is set on the recurrence curve for each site. The source recurrence plots for the case study are shown in Figure 6. From the constrained recurrence curves and the source map of the site, a suite of magnitude and distance values are randomly generated for the 100-year event, with the magnitudes consistent with the source recurrence curves and distances consistent with the site geometry. A sample distribution of 10,000 randomly generated magnitude and distance pairs consistent with a 100-year event at the site is shown in Figure 7. For more details on this method of earthquake generation see Dobossy (2006).

The generated magnitude and distance pairs are then used to simulate seismic events using the method described earlier. They are then applied to the previously described model, in OpenSees. The maximum
and minimum rotation of a connection at an interior bay and a connection at an exterior bay are recorded for each ground motion.

A distribution is fitted to the recorded data. It was found that a lognormal distribution fit the rotation data well. Figures 8 and 9 show the resulting data and distribution fit for the probability density, and probability curve for an interior connection on the second floor. Similar curves are generated for all floors and connection types. The data and probability of interest is the tail of the distribution, rotations greater than 0.025 radians. As can be seen in Figure 9, the lognormal distribution fit correlates well to the tail of the distribution.

Scenario (bin) Method

In many cases, the detailed seismic hazard information necessary to apply the “site characteristic method” may not be available. In these cases, a different approach must be taken to assess the seismic demand on the structure. One such approach is a scenario based approach, where one or more ranges of event magnitudes and distances are studied. For example one might ask, what is the structural demand for the prototype structure, given an earthquake of magnitude 7.0-7.5 at a distance of 20-30km. This can also be thought of as looking at a specific magnitude and distance bin. In this way, one can study specific types of events which a structure may see during its service life. In order to demonstrate generation of a demand curve, the forementioned example will be used.

First, a number of random magnitude and distance pairs are chosen, uniformly distributed over the mag-
magnitude and distance range of interest. Figure 10 shows the 10,000 pairs that were used for this case study. As with the “site characteristic method”, these magnitude and distance pairs are then used to generate synthetic earthquakes applied to the structural model in OpenSees.

Next, a distribution is fitted to the data. When ranges of magnitude and distance are relatively narrow, the Weibull distribution typically fits well. Figures 11 and 12 show the probability density and cumulative probability curves for the data produced in the example, along with the Weibull fit. As with the site characteristic method, the fit correlates well to the tail of the distribution which is the area of greatest interest.

To determine the reliability of a structure under seismic loading, one needs to know the demand placed on the structure in relation to the structure’s capacity. Capacity is the point at which a given structural response will cause some limit state to be reached in the structure. In some cases, the capacity of a given structural response may be a known value, however in many cases, uncertainty is involved. The response limit, or capacity, will be a single point for situations lacking uncertainty, and will be characterized by a probability distribution otherwise.

For the case study, the limit state of interest is post-tensioning strand yielding, and the measured response is the gap rotation whose demand was determined in the previous section. As floors of a SC-MRF are pushed laterally, in a seismic event, gap opening at the connections causes an expansion of the floor system, as shown in Figure 13. As the floor expands, the post-tensioning strands are stretched. A closed
form solution was derived which shows the average floor gap relative rotation at which strand yielding takes place, $\theta_{r,s}$:

$$\theta_{r,s} = \frac{N_s (t_y - t_o)}{2d_2} \frac{k_b + k_s}{k_b k_s}$$

where $N_s$ is the number of strands, $t_y$ is the force in one strand when yielding takes place, $t_o$ is the initial (average) post-tensioning force per strand, $d_2$ is the distance from the center of rotation to the beam centerline (Figure 1), $k_b$ is the axial stiffness of the beam and $k_s$ is the axial stiffness of the strands (Garlock et al., 2007a).

![Figure 13: Expanding nature of SC-MRF floors.](image)

The two main sources of uncertainty for strand yielding capacity are the yield force, $t_y$ (material uncertainty) and the original post-tensioning force, $t_o$ (construction uncertainty). Current data on steel yield strength variability shows that steel yield stress for 50ksi steel is typically normally distributed with a mean of 50.4ksi, and a standard deviation of 4.53ksi (State, 2000). No data was available for very-high-strength steel used for post-tensioning strands, so the values were extrapolated from the FEMA report for this example. The nominal ultimate stress for the strands is 270ksi. It is assumed that the yield stress is equal to 90% of the ultimate stress. Thus, the distribution of strand yield stress was assumed to be normally distributed, with a mean of 244.9ksi and standard deviation of 22.01ksi. No data was found for the uncertainty in post-tensioning at a construction site, so for this example it was assumed that given a target post-tensioning force, $t_o$, the distribution of the post-tensioning force was normal with mean $t_o$ and standard deviation $0.05t_o$. In this simple example, using the stiffness, post-tensioning, and geometry values for the second floor, yields a normal distribution with a mean $\mu$ of 0.0478 radians and a standard deviation $\sigma$ of 0.0085 radians, as shown in Figure 14.

![Figure 14: CDF of tensioning strand yielding in terms of average floor joint rotation.](image)

**OVERALL RELIABILITY**

Once the capacity and demand distributions have been determined, determining the probability of limit state attainment for the given demand parameters (i.e., corresponding to the one-hundred year earthquake
at the building site) is done by convolving the distributions of capacity and demand. The probability that the limit-state capacity is less than the demand is the “probability of failure”:

\[
P[\text{Cap} < \text{Dem}] = \sum_{i} P[\text{Cap} < \text{Dem}_i] \cdot P[\text{Dem}_i]
\]  

(9)

where \( P[\text{Cap} < \text{Dem}_i] \) is the value of the cumulative distribution of the capacity distribution at \( \theta_{r,i} \) (see Figure 14), and \( P[\text{Dem}_i] \) is the probability of a given connection rotation \( \theta_{r,i} \) (derived from the demand PDF, Figure 8). In the case of the “site characteristic method”, the probability of reaching the limit state was calculated, with results shown in Table 1.

From the results, it can be seen that more than just the limit state probability is found. One can also look at the relative risk of failure at each floor, and change the design accordingly. For example, the 6-story prototype structure has half the risk of strand yielding at the fourth and sixth floors compared to the other floors. Data such as this can guide design so that no floor is over or under designed.

Table 1: Probability of limit state attainment in a 6-story prototype structure for the 100-year quake at the hypothetical site shown in Figure 4.

<table>
<thead>
<tr>
<th>Floor</th>
<th>P[Cap &lt; Dem]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0523</td>
</tr>
<tr>
<td>2</td>
<td>0.0505</td>
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<tr>
<td>3</td>
<td>0.0508</td>
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<tr>
<td>4</td>
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<td>5</td>
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<td>6</td>
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**SUMMARY AND CONCLUSIONS**

A method was proposed for assessing structural limit state probabilities, conditional on a specified site seismic risk level, for a SC-MRF structural system. Two methods were presented for determining the limit state demand curve, one for a site where the seismic sources are well known and fully characterized, and one conditioned by a given class of seismic events specified by a range of magnitudes and distances. Due to the complex nature of a SC-MRF, Monte Carlo simulation was utilized to generate the demand curves. Both of these methods could also be used as part of a parametric study, to see how changes in the structure affect the demand. Next, a sample capacity curve was generated for post-tensioning strand yielding, using a closed form solution. While such a closed form solution worked for this simple example, simulation could be used for more complex limit states such as beam local buckling, or energy dissipation device limit state. The demand and capacity curves were then used to determine the probability of limit state exceedance at each floor of the structure.

While the case study used in this paper is a SC-MRF system at a hypothetical site, one could easily apply this method to any structural system, and with enough information about the seismic hazard, at an actual building site.

**REFERENCES**


