



A STUDY ON PUSHOVER ANALYSIS OF LOW-RISE REINFORCED CONCRETE WALL FILLED WITHIN FRAME STRUCTURE

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ABSTRACT

This paper focuses on the pushover analysis of the low-rise reinforced concrete (RC) wall filled within the frame structure. Incorporating with equilibrium and compatibility conditions, the softened model of concrete as well as the elastoplastic model of the reinforcement was taken into account for the analysis of the contribution of the RC wall, based on the fixed angle softened truss model. Accordingly, the load-deformation relation in shear of the wall subjected to monotonic lateral load can be analyzed through the proposed procedure. In order to simplify the framed wall structural model, the equivalent structural strut simulating the action of the RC wall infilled, based on the relation obtained, was modeled structurally along the diagonal of the frame for the sequential pushover analysis.

To testify the accuracy of the proposed approach, the reported results of the cyclic loading test for sixteen specimens of the structure discussed were adopted for necessary investigation. It shows that this study can provide an acceptable result of the pushover analysis for the frame structure infilled with the low-rise RC wall. Furthermore, the proposed simplified model of the equivalent structural strut might help the engineers do the structural design or evaluation easier.

Keywords: fixed angle softened model, equivalent diagonal structural strut

INTRODUCTION

The frame structure infilled with the low-rise RC wall, excited by the lateral load, incorporates essentially the bending action of the frame and the shear behavior of the RC wall, the pushover analysis of which is highly sensitive to the structural nonlinearity of those two components accordingly. Generally speaking, the analysis can be performed through the finite element method (FEM) with the shell element simulating the RC wall and the beam-column element modeling the frame structure. However, two major shortcomings give the engineers the less desire of using FEM. First, the shear performance of the RC wall is difficult to be simulated with the shell element precisely. The appropriate defining of the material property and the optimal arrangement of the element mesh still have room for further investigation, which gives a question of whether the analytical result is acceptable or not. Secondly, two different structural components required make complicated efforts in structure modeling and consume more computer time in analysis, which decreases the analytical efficiency.

Since the low-rise RC wall reveals an obvious shear behavior yielding a sufficient lateral stiffness, relative to the frame structure, it almost dominates the whole structural performance. Incorporating with equilibrium and compatibility conditions, the present paper proposes a realistic procedure using the fixed angle softened model of concrete (Pang and Hsu, 1996) to evaluate the relation of load-

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deformation in shear of the low-rise RC wall. Based on that, the lateral stiffness of the RC wall can be determined and the equivalent structural strut located at the diagonal of the frame structure can be obtained accordingly. As a result, the structure model can be simplified as the combination of the frame and the equivalent strut so that the pushover analysis can be carried out with ease. Through the analytical result, the contributions of the frame and the RC wall that includes three important components of the concrete, the transverse reinforcement and the longitudinal reinforcement, etc. can be traced in detail during each load-step of the pushover procedure. The results obtained were found to be in a good agreement with the cyclic loading test results. It shows that the proposed procedure can not only give a simplified structure model for pushover analysis with ease but also provide an acceptable result, which might help the engineers do the structural design or evaluation.

ANALYSIS PRINCIPLES OF RC WALL

Equilibrium equations

The stresses for the concrete of an RC wall element can be expressed by $(\sigma_{lc}, \sigma_{tc}, \tau_{ltc})$ shown in Fig. 1(a). In which, σ_{lc} and σ_{tc} are the normal stresses in l (longitudinal direction) and t (transverse direction), respectively, of the element, and τ_{ltc} the shear stress. The corresponding principal stresses are designated as σ_d (negative value) in compression and σ_r (positive value) in tension shown as Fig. 1(b). The angle θ between d -axis and t -axis represents the principal direction. Based on the theory of fixed angle softened truss model, the angle of cracks in the postcracking concrete coincides with the angle θ so that the equilibrium can be expressed as Eqs. 1 to 3 according to the Mohr circle of stress,

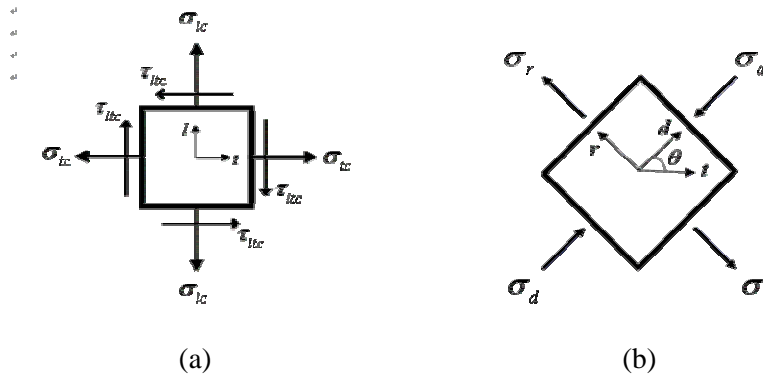


Figure 1. Stress and principal stresses of the concrete for an RC wall element

$$\sigma_{lc} = \sigma_d \cos^2 \theta + \sigma_r \sin^2 \theta \quad (1)$$

$$\sigma_{tc} = \sigma_d \sin^2 \theta + \sigma_r \cos^2 \theta \quad (2)$$

$$\tau_{ltc} = (-\sigma_d + \sigma_r) \sin \theta \cos \theta \quad (3)$$

Where, τ_{ltc} can be regarded as the average shear stress of concrete, therefore the corresponding shear force can be written as

$$V_c = \tau_{ltc} \times b_w \times d \quad (4)$$

b_w and d are the width and the effective depth, respectively, of the wall.

On the other hand, the shear force provided by the transverse reinforcement can be written as

$$V_s = A_{st} \times f_t \times \frac{d}{s} \tan \theta \quad (5)$$

Where, A_{st} , f_t and s are the cross sectional area, the stress and the spacing, respectively, of the transverse reinforcement. Therefore, the total shear force applied on the RC wall can be expressed as

$$V = V_c + V_s = \tau_{ltc} \times b_w \times d + A_{st} \times f_t \times \frac{d}{s} \tan \theta \quad (6)$$

Compatibility equations

The strains for the concrete of an RC wall element can be expressed by $(\varepsilon_{lc}, \varepsilon_{tc}, \gamma_{ltc})$ shown in Fig 2. In which, ε_{lc} and ε_{tc} are the normal strains in l (longitudinal direction) and t (transverse direction), respectively, of the element, and γ_{ltc} the shear strain. The principal strains are designated as ε_d (negative value) in compression and ε_r (positive value) in tension. Based on the theory of fixed angle softened truss model, the compatibility can be expressed as Eqs. 7 to 9 according to the Mohr circle of strain,

$$\varepsilon_l = \varepsilon_d \cos^2 \theta + \varepsilon_r \sin^2 \theta \quad (7)$$

$$\varepsilon_t = \varepsilon_d \sin^2 \theta + \varepsilon_r \cos^2 \theta \quad (8)$$

$$\gamma_{lt} = 2(-\varepsilon_d + \varepsilon_r) \sin \theta \cos \theta \quad (9)$$

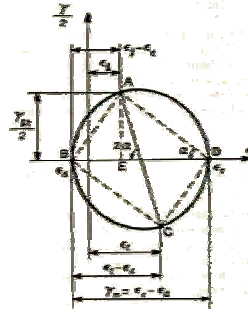


Figure 2. Mohr circle of strain of the concrete for an RC wall element

Constitute laws of concrete for softened truss model

During the past four decades, a number of tests showed that both the strength and stiffness of cracked reinforced concrete in compression are lower than that of uniaxial compressed concrete. The softening effect on the biaxial constitute laws of concrete should be taken into account.

1. Concrete in compression

The average stress-strain curve of concrete in compression (Fig. 3) was suggested as follows (Belarbi and Hsu, 1995).

$$\sigma_d = \zeta f'_c \left[2 \left(\frac{\varepsilon_d}{\zeta \varepsilon_0} \right) - \left(\frac{\varepsilon_d}{\zeta \varepsilon_0} \right)^2 \right] \quad \varepsilon_d / \zeta \varepsilon_0 \leq 1 \quad (10)$$

$$\sigma_d = \zeta f'_c \left[1 - \left(\frac{\varepsilon_d / \zeta \varepsilon_0 - 1}{2 / \zeta - 1} \right)^2 \right] \quad \varepsilon_d / \zeta \varepsilon_0 > 1 \quad (11)$$

Where f'_c is the peak stress of the standard concrete cylinder, ε_0 the corresponding strain. ζ , the peak softening coefficient, can be written as

$$\zeta = \frac{0.9}{\sqrt{1 + 600\varepsilon_r}} \quad (12)$$

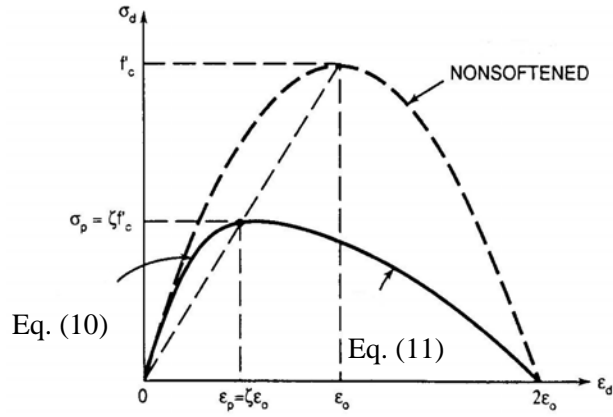


Figure 3. Compressive stress-strain relationship of softened concrete

2. Concrete in tension

The relationships of the stress-strain in tension of concrete were given by Vecchio and Collins based on their panel tests at the University of Toronto (Vecchio and Collins, 1986) and expressed as

$$\sigma_r = E_c \varepsilon_r \quad \varepsilon_r \leq \varepsilon_{cr} \quad (13)$$

$$\sigma_r = f_{cr} \left(\frac{\varepsilon_r}{\varepsilon_{cr}} \right)^{0.4} \quad \varepsilon_r > \varepsilon_{cr} \quad (14)$$

Where, E_c is the modulus of concrete = $4696\sqrt{f'_c}$ (Mpa); $f_{cr} = 0.623\sqrt{f'_c}$ and $\varepsilon_{cr} = f_{cr} / E_c \approx 8 \times 10^{-5}$ are the stress and the strain, respectively, at cracking of concrete.

Stress-strain relationship of steel

The stress-strain relationship of the reinforcement is considered as elastoplastic model

$$\sigma_s = E_s \varepsilon_s \quad \varepsilon_s \leq \varepsilon_y \quad (15)$$

$$\sigma_s = \sigma_y \quad \varepsilon_s > \varepsilon_y \quad (16)$$

Where, $E_s \approx 200Gpa$ and ε_y are the modulus and the yielding strain, respectively, of the reinforcement.

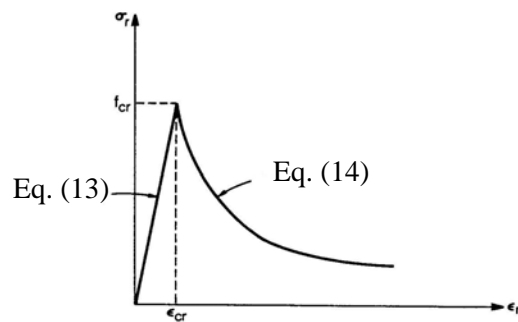


Figure 4. Tensile stress-strain relationship of concrete

EQUIVALENT DIAGONAL STRUCTURAL STRUT OF AN RC WALL

In order to simplify the framed wall structural model, the equivalent structural strut simulating the action of the RC wall infilled is modeled structurally along the diagonal of the frame for the sequential pushover analysis. A set of procedure in establishing the equivalent diagonal structural strut is proposed and described as follows:

Step 1: Input the geometric conditions including the width b_w , the depth d^* and the height h of the RC wall. The principal direction can be taken into account as the angle θ between the diagonal direction of the wall and the horizontal axis, i.e.

$$\theta = \tan^{-1}\left(\frac{h}{d^*}\right) \quad (17)$$

Step 2: Input the strength of concrete f'_c , the yielding strengths (f_{yt}, f_{yl}) together with the ratio of the cross sectional areas (ρ_t, ρ_l) of the transverse and the longitudinal reinforcements, respectively.

Step 3: Initialize the applied shear force V , the principal compressive strain ε_d and the principal tensile strain ε_t , respectively. Set their corresponding increments as $\Delta V > 0$, $\Delta\varepsilon_d < 0$ and $\Delta\varepsilon_t > 0$.

Step 4: Let $V = V + \Delta V$

Step 5: Let $\varepsilon_d = \varepsilon_d + \Delta\varepsilon_d$

Step 6: Let $\varepsilon_t = \varepsilon_t + \Delta\varepsilon_t$. If $\varepsilon_t > 0.06$, return to Step 5.

Step 7: Calculate the peak softening coefficient ζ according to Eq. 12.

Step 8: Calculate σ_d and σ_t based on Eqs. 10, 11 and Eqs. 13, 14, respectively.

Step 9: Calculate ε_l and ε_r according to Eqs. 7 and 8, respectively.

Step 10: Calculate τ_{lc} and V_c according to Eqs. 3 and 4.

Step 11: Calculate f_t based on the ε_l calculated and Eqs. 15, 16.

Step 12: Calculate V_s according to Eq. 5. Compute $\bar{V} = V_c + V_s$.

Step 13: If $|\bar{V} - V| \leq \textit{tolerance}$, calculate γ_{lc} according to Eq. 9 and the shear displacement δ as follow:

$$\delta = \gamma_{lc} \times h \quad (18)$$

Then record V and δ . Let $\Delta V > 0$ for $\varepsilon_d / \zeta \varepsilon_0 \leq 1$, or $\Delta V < 0$ for $\varepsilon_d / \zeta \varepsilon_0 > 1$, and then return to Step 4.

If $|\bar{V} - V| > \textit{tolerance}$, return to Step 6.

Step 14: If $\varepsilon_d < -0.0035$, return to Step 5. Otherwise, stop the iteration.

As a result, the relationship between the horizontal shear force and the horizontal shear displacement, $V - \delta$, of the RC wall can be obtained through the above procedure. Based on the equilibrium and compatibility, the load-displacement relation of the equivalent structural strut along the diagonal of the frame, $V_{strut} - \delta_{strut}$, can be calculated by following transformations.

$$V_{strut} = V / \cos \theta \quad (19)$$

$$\delta_{strut} = \delta \times \cos \theta \quad (20)$$

Associated with the relation of $V_{strut} - \delta_{strut}$ for the equivalent diagonal structural strut obtained, the plastic hinge properties of the beam and column can be set according to the authors' previous study (Sung et al., 2005), and the sequential pushover analysis of the frame structure infilled with the low-rise RC wall can then be carried out with ease.

CASE STUDIES

The reported results of the cyclic loading test for sixteen specimens of the structure discussed were served as case studies. The detail data of the specimens are listed in Table 1. (Benjamin and Williams, 1957), (Hirosawa, 1975), (Yamada, 1974)

Table 1 Specimen data of the low-rise framed RC Walls

No.	Specimen	ρ_l	ρ_t	h (cm)	d^* (cm)	t_w (cm)	f'_c (Mpa)	f_{yl} (Mpa)	f_{yt} (Mpa)	Beam (cm×cm)	Column (cm×cm)
1	4b2-3	0.005	0.005	56	122	5	19.5	359	359	13×10	13×10
2	4b2-4	0.005	0.005	56	178	5	26.4	359	359	13×10	13×10
3	140-1	0.0066	0.0066	180	396	12	20.6	572	572	36×36	36×36
4	141-2	0.0066	0.0066	180	396	12	20.8	572	572	36×36	36×36
5	142-3	0.0066	0.0066	180	396	12	21.3	572	572	36×36	36×36
6	143-4	0.0033	0.0033	180	396	12	19.6	572	572	36×36	36×36
7	144-5	0.0033	0.0033	180	396	12	20.8	572	572	36×36	36×36
8	145-6	0.0069	0.0066	180	396	12	20.5	284	284	36×36	36×36
9	146-7	0.0069	0.0066	180	396	12	19.6	284	284	36×36	36×36
10	147-8	0.0077	0.0074	180	396	12	20.9	397	397	36×36	36×36
11	t=40 ρ=0.0031	0.0031	0.0031	60	133	4	35.6	286	286	13×13	13×13
12	t=40 ρ=0.0063	0.0063	0.0063	60	133	4	30.4	286	286	13×13	13×13
13	t=40 ρ=0.0126	0.0126	0.0126	60	133	4	31.5	286	286	13×13	13×13
14	t=30 ρ=0.0084	0.0084	0.0084	60	133	3	32.8	286	286	13×13	13×13
15	t=20 ρ=0.0063	0.0063	0.0063	60	133	2	30.1	286	286	13×13	13×13
16	t=20 ρ=0.0126	0.0126	0.0126	60	133	2	33.7	286	286	13×13	13×13

To make a shorter illustration, we only listed the detailed analytical results of the specimen 143-6 here. Figs. 5 and 6 give the stress-strain relation obtained of concrete for the RC wall. Figs. 7 and 8 give the stress-strain relation obtained of the reinforcements. It can be seen that the yielding of the transverse reinforcement comes later than that of the longitudinal reinforcement, and soon after that the wall reaches ultimate eventually. Fig. 7 shows the relationship between shear stress and shear strain obtained of the RC wall. Fig. 8 gives the developed shear strength of the RC wall. In which, the contributions of shear strength both from concrete and the transverse reinforcement can be traced in detail. It shows that the transverse reinforcement begins to play an active role in sharing the shear force soon after the development of the peak strength of the concrete. The less the descending concrete strength has, the more the transverse reinforcement strength contributes. The situation goes on until the presence of the ultimate strain of the concrete. Fig. 9 shows the load-displacement relation obtained for the equivalent strut which is employed to define the structural nonlinearity of the strut located at the diagonal of the frame structure. Finally, the result of the pushover analysis can be obtained and compared with the experimental one (Fig. 10).

Similarly, the pushover analyses of all the specimens can be carried out based on the proposed procedure and their precisions can be discussed through the comparisons with the experimental results. Fig. 10 gives all the consequences. The analyses give an over-estimation of the shear strength for the cases of the specimens of No.1, No. 7, No. 15 and No. 16, while a under-estimation for No. 2, No. 3, No. 4, No.5, No.9, No. 10 and No. 13. As for the cases of No. 6, No. 8, No. 9, No. 11, No.12 and No. 14, the analyses give a good prediction of the shear strength. It is noteworthy that only the parameter of f'_c , the compressive strength of concrete, is a little bit different in the cases of No. 3, No. 4, No.5, but it appears a large difference among the experimental results. This unreasonable outcome might be caused by the experimental deviations. The statistic of the analyses shows that the structural performance of the low-rise RC wall could be predicted theoretically based on the proposed procedure. As a result, the action of the RC wall can be modeled structurally as an equivalent diagonal strut within the frame structure so that the pushover analysis can be carried out with ease.

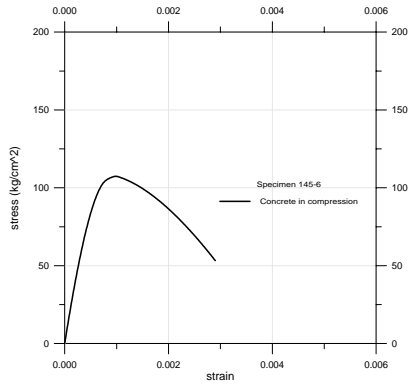
CONCLUSIONS

The following conclusions can be drawn after the investigations.

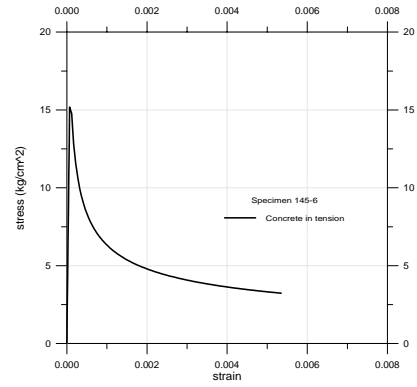
- (1) Based on the fixed angle softened truss model, the structural performance of the low-rise RC wall excited by the monotonic lateral force can be predicted theoretically through the proposed procedure. The contributions of the concrete, the longitudinal reinforcement and the transverse reinforcement can be traced, respectively.
- (2) The action of the RC wall can be modeled structurally as an equivalent strut located at the diagonal of the frame structure within an acceptable precision, based on the proposed procedure, so that the pushover analysis of the frame structure infilled with the RC wall can be carried out with ease, which might give a great convenience to the engineers in performing structural design or evaluation easier.
- (3) It shows that this study can provide a reasonable result of the pushover analysis for the frame structure infilled low-rise RC wall.

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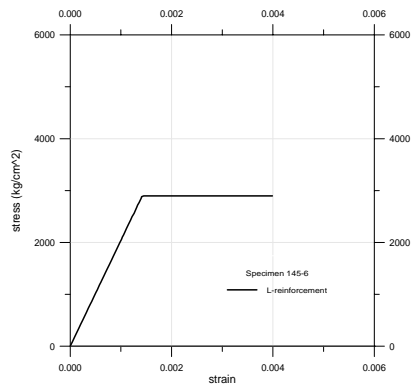


(a) compressive

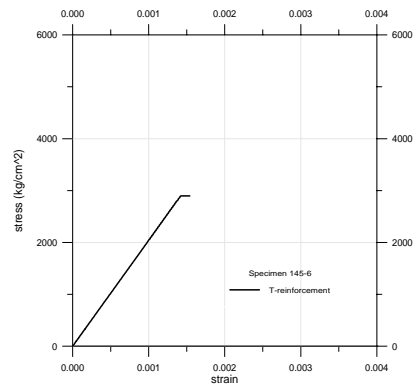


(b) tensile

Figure 5. Developed stress-strain relationship of concrete (specimen 143-6)



(a) longitudinal



(b) transverse

Figure 6. Developed stress-strain relationship of reinforcement (specimen 143-6)

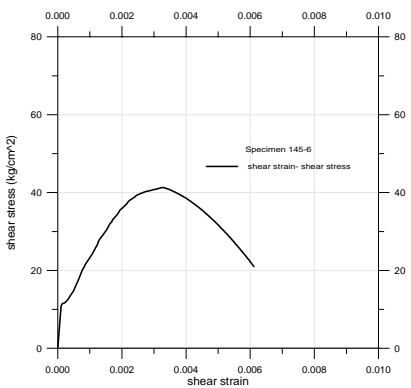


Figure 7. Developed shear stress – shear strain relationship (specimen 143-6)

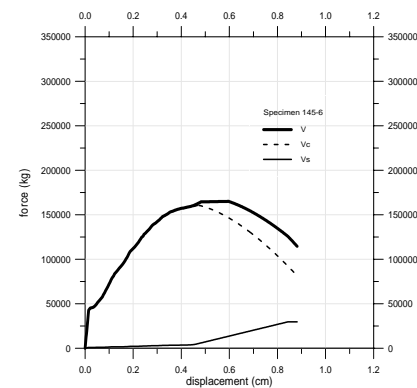


Figure 8. Shear strength of RC wall (specimen 143-6)

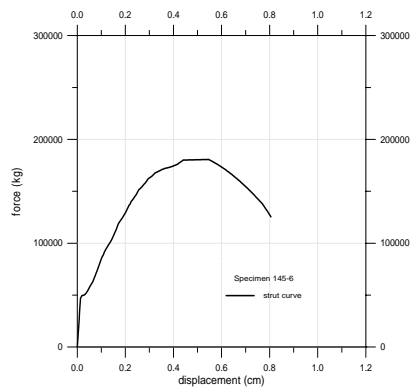


Figure 9. Load-displacement of equivalent strut (specimen 143-6)

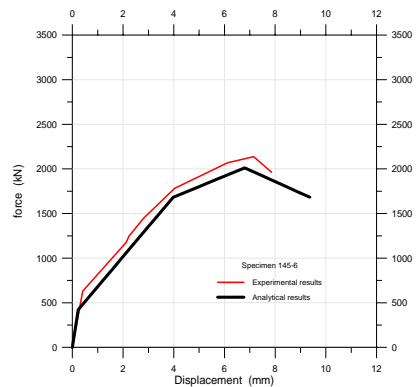


Figure 10. Shear strength of RC wall (specimen 143-6)

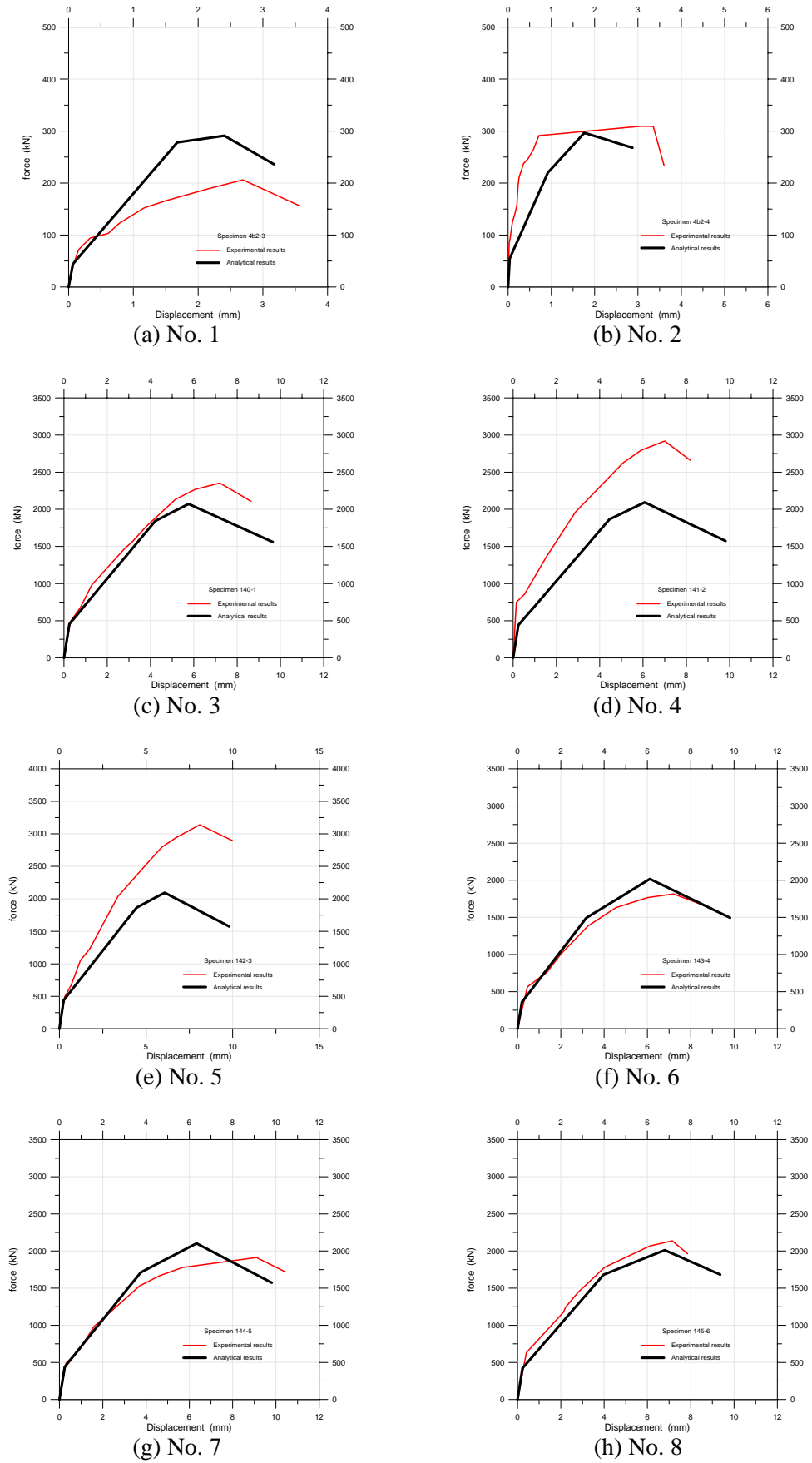
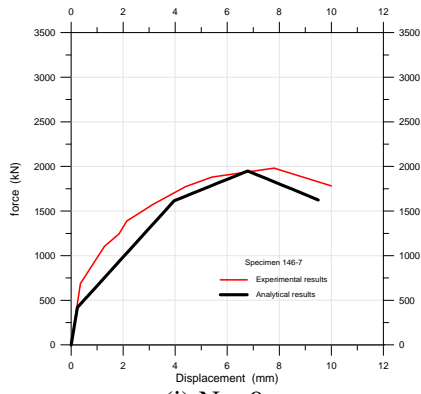
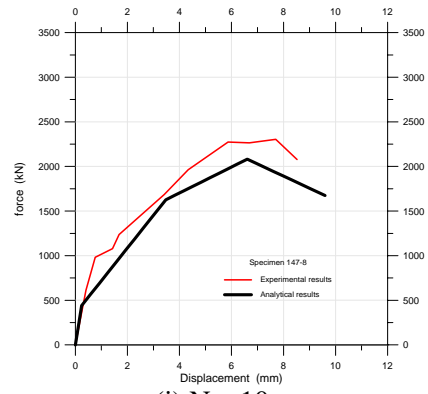


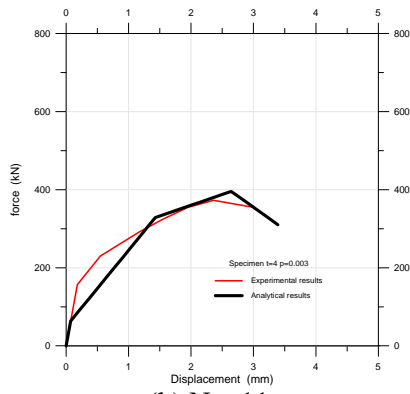
Figure 11. Comparison between pushover analysis result and experimental result



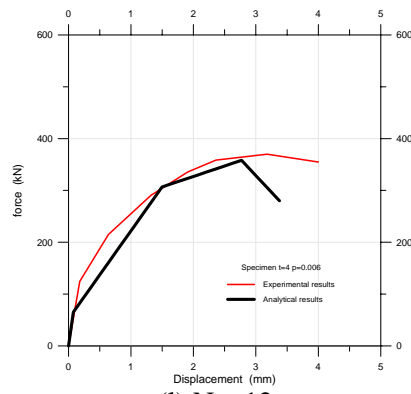
(i) No. 9



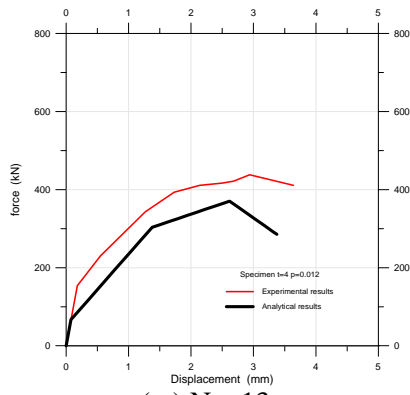
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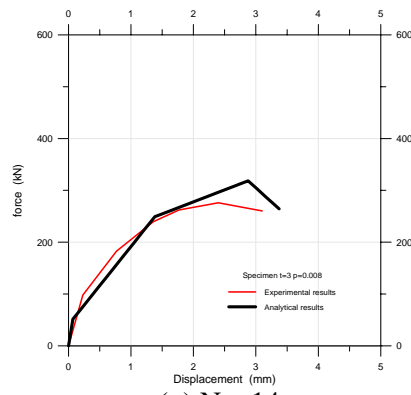
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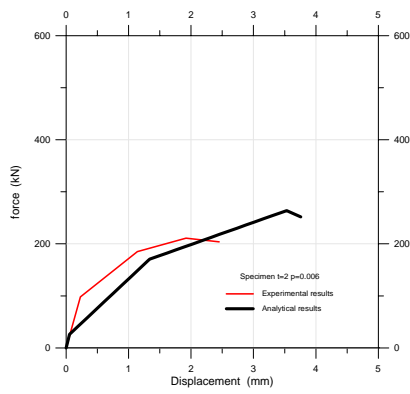
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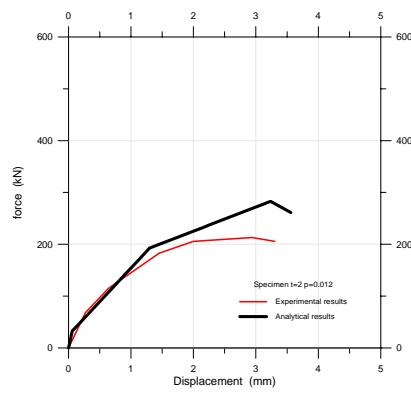
(m) No. 13



(n) No. 14



(o) No. 15



(p) No. 16

Figure 11. Comparison between pushover analysis result and experimental result (Continued)