AN ENERGY DISSIPATION BASED SEMI-ACTIVE CONTROL OF CIVIL STRUCTURES WITH A NEW MECHATRONIC VIBRATION CONTROL DEVICE

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Abstract

We propose a new mechatronic vibration control device (VCD) for vibration suppression of civil engineering structures. The VCD consists of an electric motor that can generate a variable damping force and a moving mass that produces a large inertia force which is proportional to the relative acceleration between two floors of the structure. The variable damping property of the VCD is realized by changing the value of an electric resistance connected to the electric motor. With the capability of the variable damping a simple semi-active control strategy based on Lyapunov method is adopted. We discuss issues about an appropriate selection of the Lyapunov matrix and the performance analysis of the semi-active system based on the theory on dissipative dynamical systems. A simulation result of NCREE benchmark building with the proposed VCD shows the effectiveness of the present hardware and the semi-active control technique.

Keywords: Mechatronic vibration control device, Semi-active control, Lyapunov function, Dissipation theory

1 INTRODUCTION

Various methodologies for vibration control of civil structures have been proposed by many researchers. The traditional scheme was passive vibration control, i.e., dissipation of the vibration energy to the outside of the structural systems with (dynamic) dampers. Passive control is quite simple, however it has some limitations, e.g., poor performance robustness to an uncertainty or a change of the dynamic characteristic of the structures, and difficulty and complexity in tuning dampers when multi-mode of vibration, e.g., 1st and 2nd modes of vibration should be suppressed simultaneously.

Active vibration control technique is a candidate for a breakthrough to overcome the above problems of passive control and has been studied (Spencer et al. (1998) and the references therein) extensively these decades. Although active control methodology can resolve most of the above drawbacks of passive control, active control requires a large energy source to produce the active damping force and this fact has been an obstacle in applying active methods to general vibration control problems.

Semi-active control, which is not necessarily new (Karnopp et al., 1974) either, can be recognized as an intermediate between passive and active schemes in the sense of not only the performance on vibration suppression but also the complexity of the control system. In civil structures semi-active control technique is getting more realistic recently along with the development of a large scale dampers whose damping property is able to be changed (Sodeyama et al., 2004). In most semi-active control vibration suppression is achieved by changing the damping coefficient of the semi-active damper. The damping property of the semi-active damper is changed based on the following two strategies, i.e., an active controller is synthesized with a standard control theory (e.g., sky-hook, LQ or $\mathcal{H}_\infty$ control, etc.) firstly,
and the semi-active damper works to emulate the targeted active control as much as possible (Karnopp et al., 1974; Nishimura et al., 2001); the variable damping property of the semi-active damper is assumed from the very beginning of the control system design (Gavin, 2001).

In this paper we propose a new mechatronic vibration control device (VCD) for vibration suppression of civil engineering structures. The VCD consists of an electric motor that can generate a variable damping force and a moving mass that produces a large inertia force which is proportional to the relative acceleration between two floors of the structure. The variable damping property of the VCD is realized by changing the value of an electric resistance connected to the electric motor. With the capability of the variable damping a simple semi-active control strategy based on Lyapunov method (Gavin, 2001) is adopted. We discuss issues about an appropriate selection of the Lyapunov matrix and the performance analysis of the semi-active system based on the theory on dissipative dynamical systems. A simulation result of NCREE benchmark building with the proposed VCD shows the effectiveness of the present hardware and the semi-active control technique.

The notation is standard: \( \mathcal{R}^{m\times n} \) denotes the set of \( m \times n \) real matrices and \( S^m \) denotes the set of \( m \)-dimensional symmetric real matrices. \( t, s, L(\bullet(t)) \), \( I \), \( 0_{m\times n} \) and \( ^t \) denote time, Laplace operator and Laplace transformation of \( \bullet(t) \), an identity matrix with a conformable dimension, \( m \times n \) zero matrix and the transpose of a matrix respectively. The \( L_2 \) norm of a signal and the \( \mathcal{H}_\infty \) norm of a linear time invariant (LTI) system are denoted by \( \| \bullet \| \) and \( \| \bullet \|_\infty \) respectively.

## 2 MODELING OF STRUCTURAL SYSTEMS

### 2.1 Principle of the Vibration Control Device (VCD)

We propose a new mechatronic vibration control device (VCD) for vibration suppression of civil engineering structures. The schematic diagram of the VCD is shown in Fig. 1. The VCD is assumed to be installed between two floors of a structure as conventional dampers. In Fig. 1, \( q_i(t) \) \( (i = 1, 2) \), \( VCD \) and \( d^{VCD}(t) \) are a displacement of each floor, an equivalent mass and a time-varying damping coefficient of the VCD respectively. The force produced by the relative motion of two floors is denoted by \( f^{VCD}(t) \).

The VCD consists of an electric motor that can generate a variable damping force and a moving mass that produces a large inertia force which is proportional to the relative acceleration between two floors of the structure. The variable damping property of the VCD is realized by changing the value of an electric resistance connected to the electric motor. The detailed structure and the dynamical property of the VCD is also presented in Matsuoka et al. (2006). From the control engineering point of view the input signals of VCD are the relative acceleration and the velocity between two floors and the output is the force \( f^{VCD}(t) \) described by

\[
f^{VCD}(t) = m^{VCD}(\ddot{q}_2(t) - \ddot{q}_1(t)) + d^{VCD}(t)(\dot{q}_2(t) - \dot{q}_1(t)).
\]

### 2.2 Model of Civil Structures with VCD

Let us consider a structure which is installed \( n_{VCD} \)-VCDs, whose inertial and (variable) damping coefficients are denoted by \( m_j^{VCD} \) and \( d_j^{VCD}(t) \) \( (i = 1, \ldots, n_{VCD}) \) respectively. The equation of motion of the structure is given as

\[
M \ddot{q}(t) + D \dot{q}(t) + Kq(t) = b_2 \dot{w}(t) + b_1 \dot{w}(t) + b_0 w(t),
\]

where \( q(t) \in \mathcal{R}^n \) and \( w(t) \in \mathcal{R}^{n_w} \) are the displacement and the disturbance vector of the structure respectively. Matrices \( M, D, K \in \mathcal{S}^n \) and \( b_j \in \mathcal{R}^{n \times n_{w}} \) \( (j = 0, 1, 2) \) are the mass, damping, stiffness and influence coefficient matrices, respectively. Note that matrices \( D \) and \( b_1 \) depend on the variable damping coefficient of the VCDs. The state space form of Eq. 2 becomes as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu_w(t) \\
\dot{z}(t) &= Cz(t) + Dz u_w(t)
\end{align*}
\]
where
\[ x(t) := \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix}, u_w(t) := \begin{bmatrix} w(t) \\ \dot{w}(t) \end{bmatrix}, A := \begin{bmatrix} 0_{n \times n} & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, B := \begin{bmatrix} 0_{n \times 3n_w} \\ M^{-1} \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} \end{bmatrix}. \]

The vector \( z(t) \in \mathbb{R}^n \) is the output vector to evaluate the performance of the structural system with VCDs, e.g., the acceleration of each floor, the relative displacement between adjacent two floors, and the force produced by VCDs, etc.. Note that all coefficient matrices contain the variable damping of the VCDs, i.e., matrices \( A, B, C_z, D_z \) are functions on the variable damping coefficient \( d^{VCD}_i, i = 1, \ldots, n_{VCD} \) of the VCDs. In this paper we assume each variable damping coefficient of the VCD can be controlled as the following:

\[ d^{VCD}_i \leq d^{VCD}_i \leq d^{VCD}_i, \quad i = 1, \ldots, n_{VCD} \]

where \( d^{VCD}_i \geq 0 \) and \( d^{VCD}_i \geq d^{VCD}_i \) are the maximum and the minimum damping coefficient of \( i \)-th VCD.

### 3 CONTROL METHOD

#### 3.1 Lyapunov based control scheme

In this paper we employ a semi-active type control method based on Lyapunov function (Gavin, 2001) of the structural system with VCDs. The damping coefficient of each VCD is changed to produce a targeted force (if possible) when a variable damping and a variable stiffness are assumed to be possible in the structural system. Those variable damping and stiffness properties are determined for maximizing the dissipation rate of a Lyapunov function to be defined.

Furthermore we try to clarify the effectiveness of the semi-active control compared to the high-damping case using the theory of dissipation (Willems, 1972; Kokotović and Arcak, 2001), which has been used for an analysis of (possibly nonlinear) dynamical systems for a long time. Using the result of the analysis, we propose an appropriate selection of the Lyapunov matrix. The detail of the control method is presented in this section.

Let us consider a Lyapunov function \( V(t) \) given as the following:

\[ V(t) := x^T(t)Sx(t), \quad S \in \mathbb{S}^{4n}, S > 0 \]

With Eq. 3 the time derivative of \( V(t) \) is obtained by

\[ \dot{V}(t) = 2x^T(t)S\dot{x}(t) = 2x^T(t)S(Ax(t) + Bu_w(t)) = \dot{V}_i(t) + \dot{V}_v(t), \]

where \( \dot{V}_i(t) \) is a time invariant part of the time derivative of the Lyapunov function \( V(t) \) and \( \dot{V}_v(t) \) is a time varying portion of \( \dot{V}(t) \), i.e., terms containing the variable damping and the stiffness components in the present control setup.
Assume the matrix $S$ in Eq. 5 is partitioned as the following:

$$S := \begin{bmatrix} X & Z \\ Z^T & Y \end{bmatrix}, \; X, Y \in S^{2n}, \; Z \in \mathbb{R}^{2n \times 2n}$$

(7)

Then $V_c(t)$ in Eq. 6 can be described as follows:

$$\dot{V}_c(t) = - \left[ q^T(t) \; \dot{q}^T(t) \right] \left[ \begin{array}{c} Z \\ Y \end{array} \right] M^{-1} \left\{ \begin{array}{c} \dot{q}(t) \\ \dot{w}(t) \end{array} \right\} + L^\alpha_k \left[ \begin{array}{c} q(t) \\ w(t) \end{array} \right]$$

(8)

$$L^\alpha_k := \left[ D^\alpha \; b^\alpha \right] = \sum_{i=1}^{n_{VCD}} d_i^\alpha P_i^\alpha, \; L^K_k := \left[ K^\nu \; b_0^\nu \right] = \sum_{i=1}^{n_{VCD}} k_i^\nu P_i^K$$

(9)

where matrices $D^\alpha \in \mathbb{R}^{n \times n}$, $b^\alpha \in \mathbb{R}^{n \times n_w}$, $K^\nu \in \mathbb{R}^{n \times n}$ and $b_0^\nu \in \mathbb{R}^{n \times n_w}$ are the (fictitiously realized) variable damping and the stiffness part of matrices $D$, $b_1$, $K$ and $b_0$ in Eq. 2 respectively. The variable damping coefficient and the variable spring constant are defined as $d_i^\alpha$ and $k_i^\nu$ ($i = 1, \ldots, n_{VCD}$). Each range of $d_i^\alpha$ and $k_i^\nu$ ($i = 1, \ldots, n_{VCD}$) is defined as

$$d_{i-1}^\alpha \leq d_i^\alpha \leq \bar{d}_i^\alpha, \; k_{i-1}^\nu \leq k_i^\nu \leq \bar{k}_i^\nu, \; i = 1, \ldots, n_{VCD}$$

(10)

where $\bar{d}_i^\alpha$, $d_i^\alpha$, $\bar{k}_i^\nu$ and $k_i^\nu$ are the upper and lower-bounds of $d_i^\alpha$ and $k_i^\nu$ respectively. With the definition in Eqs. 8 and 9 we have another representation of $V_c(t)$ as the following:

$$\dot{V}_c(t) = \sum_{i=1}^{n_{VCD}} (d_i^\alpha \alpha_i + k_i^\nu \beta_i)$$

(11)

where

$$\alpha_i := - \left[ q^T(t) \; \dot{q}^T(t) \right] \left[ \begin{array}{c} Z \\ Y \end{array} \right] M^{-1} P_i^\alpha \left[ \begin{array}{c} \dot{q}(t) \\ \dot{w}(t) \end{array} \right] \in \mathbb{R},$$

$$\beta_i := - \left[ q^T(t) \; \dot{q}^T(t) \right] \left[ \begin{array}{c} Z \\ Y \end{array} \right] M^{-1} P_i^K \left[ \begin{array}{c} q(t) \\ w(t) \end{array} \right] \in \mathbb{R}, \; i = 1, \ldots, n_{VCD}$$

From Eqs. 10 and 11 the best selection of the variable damping and stiffness coefficients to minimize $\dot{V}(t)$, i.e., to maximize the dissipation rate of the Lyapunov function $V(t)$ are given as follows:

$$d_i^\alpha = \begin{cases} \bar{d}_i^\alpha & (\alpha_i \leq 0) \\ d_i^\alpha & (\alpha_i > 0) \end{cases}, \; k_i^\nu = \begin{cases} \bar{k}_i^\nu & (\beta_i \leq 0) \\ k_i^\nu & (\beta_i > 0) \end{cases}$$

(12)

With the variable damping and stiffness coefficients $d_i^\alpha$ and $k_i^\nu$ ($i = 1, \ldots, n_{VCD}$) determined as Eq. 12 the force $f_i^a(t)$ ($i = 1, \ldots, n_{VCD}$) to be produced by each VCD is defined as the following:

$$f_i^a(t) := m_i^{VCD} \ddot{q}_i^a(t) + d_i^\alpha \dot{q}_i^a(t) + k_i^\nu q_i^a(t)$$

(13)

where $q_i^a(t) \in \mathbb{R}$ is the relative displacement of the two floors which are installed $i$-th VCD.

### 3.2 Semi-active control with VCDs

In this paper the variable damping capability of the VCD is devoted to produce the force $f_i^a(t)$ ($i = 1, \ldots, n_{VCD}$) in Eq. 13 as much as possible. That is, the damping coefficient of $i$-th VCD is determined by the following equation:

$$m_i^{VCD} \ddot{q}_i^a(t) + d_i^{VCD} \dot{q}_i^a(t) = f_i^a(t) = m_i^{VCD} \ddot{q}_i^a(t) + d_i^\alpha \dot{q}_i^a(t) + k_i^\nu q_i^a(t), \; i = 1, \ldots, n_{VCD}$$

(14)
With Eqs. 14 and 4 the feasible value of \( d_i^{VCD} \) producing the target force (as much as possible) is given as the following:

\[
d_i^{VCD}(t) = \begin{cases} 
\frac{d_i^{VCD}}{d_i^{VCD} + k_i q'_i(t)} (f_i^q(t)/q'_i(t) > d_i^{VCD} : \text{Case I}) \\
\frac{d_i^{VCD}}{d_i^{VCD} + k_i q'_i(t)} (d_i^{VCD} \leq f_i^q(t)/q'_i(t) \leq d_i^{VCD} : \text{Case II}) , i = 1, \ldots, n_{VCD} \\
\frac{d_i^{VCD}}{d_i^{VCD} + k_i q'_i(t)} (f_i^q(t)/q'_i(t) < d_i^{VCD} : \text{Case III}) 
\end{cases}
\]  

(15)

In case II of Eq. 15 the targeted force \( f_i^q(t) \) is able to be realized correctly by the \( i \)-th VCD. Otherwise the force \( f_i^q(t) \) cannot be achieved, however, the force produced by the VCD \( (m_i^{VCD}q'_i(t) + d_i^{VCD}q'_i(t)) \) in such cases is the best approximation of \( f_i^q(t) \) in the current situation because of the saturation characteristic of \( d_i^{VCD} \) (Eq. 4). Note that even the saturation characteristic exists in the current control scheme the stability of the structural system with VCDs is guaranteed.

### 3.3 Selection of the Lyapunov matrix

An appropriate selection of the Lyapunov matrix \( S \) in Eq. 5 and an advantage of the semi-active control over the high damping and stiffness case are discussed in this subsection. We employ the theory on dissipative dynamical systems (Willems, 1972; Kokotović and Arcak, 2001) as a tool for the analysis. The theory about dissipative systems is widely used to analyze an input-output property of (possibly nonlinear) dynamical systems, e.g., circuit systems and mechanical systems, etc..

In this paper the Lyapunov matrix \( S \) in Eq. 5 is defined as the solution to a following LMI (Linear matrix inequality) optimization problem:

\[
\text{Minimize } \gamma > 0 \text{ subject to } \begin{bmatrix} \bar{A}^T S + S \bar{A} & S \bar{B} & C_z^T \\ \bar{B}^T S & -\gamma I & D_z^T \\ C_z & D_z & -\gamma I \end{bmatrix} < 0 \quad (S \in S^{4n}).
\]  

(16)

where matrices \( \bar{A} \in \mathbb{R}^{2n \times 2n} \), \( \bar{B} \in \mathbb{R}^{2n \times 3n} \), \( C_z \in \mathbb{R}^{n_0 \times 2n} \) and \( D_z \in \mathbb{R}^{n_0 \times 3n} \) are coefficient matrices \( A \), \( B \), \( C_z \) and \( D_z \) in Eq. 3 where all fictitious variable damping and the stiffness coefficients are fixed as their maximum values, i.e.,

\[
d_i^g = d_i^T, \quad k_i^g = k_i^T, \quad i = 1, \ldots, n_{VCD}.
\]  

(17)

It is well known that we can obtain the global optimal solution to the problem given as Eq. 16 quite effectively in a numerical manner and LMIs are widely utilized mainly in control community for control synthesis and analysis (Boyd et al., 1994). The positive scalar \( \gamma > 0 \) is the \( \mathcal{H}_\infty \) norm (Scherer et al., 1997) of the transfer function \( G(s) := D_z + C_z(sI - A)^{-1}B \) defined as the following:

\[
\gamma := \|G(s)\|_\infty := \sup_{\|u_w(t)\| < \infty} \|z(t)\| = \|z(t)\|_{\infty}
\]  

(18)

Note that for time varying and/or nonlinear systems the definition in Eq. 18 is still valid and the index \( \gamma \) is called \( \mathcal{L}_2 \) gain. From Kokotović and Arcak (2001) the condition \( \|G(s)\|_\infty \leq \gamma \) is equivalent to the following inequality condition on \( V(t) \):

\[
h(t) := V(t) + \gamma^2 u_w^T(t)u_w(t) - z^T(t)z(t) = V_i(t) + \bar{V}_i(t) + \gamma^2 u_w^T(t)u_w(t) - z^T(t)z(t) \leq 0, \quad \forall t, \quad \|u_w(t)\| < \infty, \quad z(t)
\]  

(19)

Note that there is at least one combination of \( u_w(t) \) and \( z(t) \) which yields \( h(t) = 0 \).

To show the advantage of the semi-active control over the high damping and stiffness case let us define a set \( C_z \) defined as the following:

\[
C_z := \{u_w(t), z(t)|h(t) = 0\}
\]  

(20)
The set $\mathcal{C}_e$ is made by gathering all critical combinations of the input $u_w(t)$ and the output $z(t)$, i.e., $h(t) = 0$ in Eq. 19. From the above discussion the set $\mathcal{C}_e$ does never become empty if the damping and the stiffness of the structural system are fixed their maximum. If we can change the damping and the stiffness according to Eq. 12 we can avoid some critical cases ($h(t) = 0$), that is, the critical combinations of $u_w(t)$ and $z(t)$ by switching $d_i^v = d_i^p$ (if $\alpha_i > 0$) or $k_i^p = k_i^v$ (if $\beta_i > 0$). In such cases we can exclude some elements from the set $\mathcal{C}_e$ by changing $V_{\text{CD}}(t)$ in Eq. 19, i.e., for some $\{u_w(t), z(t)\} \in \mathcal{C}_e$ we can change the equality $h(t) = 0$ into an inequality $h(t) < 0$. On the other hand we can regard the system behaves as the high damping and stiffness case, i.e., $h(t) = 0$ for $\{u_w(t), z(t)\} \in \mathcal{C}_e$ if $\alpha_i \leq 0$ and $\beta_i \leq 0$ ($\forall i = 1, \ldots, n_{\text{VCD}}$) in Eq. 12. From the above observation in the sense of $L_2$ gain of the structural system, we can conclude that changing the damping and the stiffness like Eq. 12 guarantees not only the same performance as the high damping and stiffness case, but also we can expect a better performance being achieved.

Of course the degree of the performance improvement of the variable damping and the stiffness scheme highly depends on how the disturbance $u_w(t)$ is. We can easily imagine a case that for a particular disturbance the performance of the semi-active control is much better than that of the high damping and stiffness case even the amount of the improvement is very small for another disturbance under the same control law (given by Eq. 12). However the result of the analysis is quite interesting because we have not had a good explanation for a problem why the semi-active control scheme is better than that of the passive high damping (and/or stiffness) from a theoretical aspect.

Furthermore we can conclude that the selection of the Lyapunov matrix in this paper is valid since we can discuss the disturbance attenuation property of the structural system using Eqs. 12 and 19, etc..

## 4 NUMERICAL EXAMPLE

We consider NCREE 3-story benchmark building with VCDs as an example. The model of the benchmark building is shown in Fig. 2. The structural parameters of the benchmark building are given in Table 1. Between all neighboring floors VCDs are installed (denoted by VCD, $i = 1, 2, 3$) whose mass and damping are $m_i^{\text{VCD}}$ and $d_i^{\text{VCD}}$ ($i = 1, 2, 3$) respectively. By letting $q(t) := [q_1(t)\ q_2(t)\ q_3(t)]^T$ in Eq. 2 the coefficient matrices of the equation of motion are given as follows:

$$M := \begin{bmatrix}
m_1 + m_1^{\text{VCD}} + m_2^{\text{VCD}} & -m_2^{\text{VCD}} & 0 \\
-m_2^{\text{VCD}} & m_2 + m_3^{\text{VCD}} + m_3^{\text{VCD}} & -m_3^{\text{VCD}} \\
0 & -m_3^{\text{VCD}} & m_3 + m_3^{\text{VCD}}
\end{bmatrix},$$

$$D := \begin{bmatrix}
d_1 + d_1^{\text{VCD}} + d_2^{\text{VCD}} + d_4 \\
d_2 - d_2^{\text{VCD}} \\
d_2 - d_2^{\text{VCD}} - d_4
\end{bmatrix},$$

$$K := \begin{bmatrix}
k_1 + k_2 + k_4 & -k_2 & -k_3 \\
-k_2 & k_2 + k_3 & -k_3 \\
-k_3 & -k_3 & k_3 + k_4
\end{bmatrix},$$

$b_2 := \begin{bmatrix}
m_1^{\text{VCD}} \\
0 \\
0
\end{bmatrix}, b_1 := \begin{bmatrix}
d_1 + d_1^{\text{VCD}} \\
0 \\
0
\end{bmatrix}, b_0 := \begin{bmatrix}
k_1 \\
0 \\
0
\end{bmatrix}$

In this example the output vector $z(t)$ (in Eq. 3) for the performance evaluation is defined as

$$z(t) := [q(t)\ \dot{q}(t)\ \ddot{q}(t)\ q_r(t)\ \dot{q}_r(t)\ \ddot{q}_r(t)\ f_{\text{VCD}}^e(t)\ f_{\text{D}}^e(t)]^T,$$

where

$$q_r(t) := \begin{bmatrix}
q_1(t) - w(t) \\
q_2(t) - q_1(t) \\
q_3(t) - q_2(t)
\end{bmatrix},$$

$$f_{\text{VCD}}^e(t) := f_{\text{VCD}}^e(t) + f_{\text{D}}^e(t),$$

$$f_{\text{VCD}}^e(t) := \begin{bmatrix}
m_1^{\text{VCD}}(\dot{w}(t) - \dot{q}_1(t)) \\
m_2^{\text{VCD}}(\dot{q}_2(t) - \dot{q}_1(t)) \\
m_3^{\text{VCD}}(\dot{q}_3(t) - \dot{q}_2(t))
\end{bmatrix},$$

$$f_{\text{D}}^e(t) := \begin{bmatrix}
d_1^{\text{VCD}}(\ddot{w}(t) - \ddot{q}_1(t)) \\
d_2^{\text{VCD}}(\ddot{q}_2(t) - \ddot{q}_1(t)) \\
d_3^{\text{VCD}}(\ddot{q}_3(t) - \ddot{q}_2(t))
\end{bmatrix}.$$
Note that \( q_i(t) \) is the relative displacement vector between two neighboring floors and the \( i \)-th component of \( f^i_{VCD}(t) \) is the total force produced by \( i \)-th VCD \((i = 1, 2, 3)\). The force \( f^i_1(t) \) and \( f^D_i(t) \) are the inertial and the (variable) damping portions of \( f^i_{VCD}(t) \) respectively. The equivalent mass of VCDs, the maximum and the minimum values of \( d_{VCD}^i \) in Eqs. 1 and 10 are determined as follows:

\[
m_{VCD}^i = 2m_i, \quad d_{VCD}^{\max}_{i} := 0, \quad d_{VCD}^{\min}_{i} := 2\zeta_i \sqrt{m_i k_i}, \quad i = 1, 2, 3
\]

where \( \zeta_1 = 0.6, \zeta_2 = \zeta_3 = 0.2 \) respectively. We set the the maximum and the minimum values of (fictitious) variable damping and the stiffness Eq. 17 to create the targeted force \( f^i_a(t) \) \((i = 1, 2, 3)\) in Eq. 13 as follows:

\[
d^V_{i} := d^V_{iVCD}, \quad d^D_{i} := d^D_{iVCD}, \quad k^V_{i} = k^D_{i} = 0, \quad i = 1, 2, 3
\]

Note that we do not introduce the variable stiffness property in this example.

As the earthquake disturbance a scaled El Centro NS wave whose maximum acceleration equals to 2.0 \([m/s^2]\) is adopted. The time history of the disturbance is shown in Fig. 3. Following three responses of the benchmark structure are obtained: NC: Without VCDs; HD: With VCDs (Passive). All damping coefficients of VCDs are fixed their maximum values.; SA: With VCDs (Semi-active control). Each damping of the VCD is changed according to Eq. 15. The absolute displacement and the acceleration of each floor is shown in Fig. 4. The blue, green and red lines denote the response without VCDs (NC), with VCDs whose damping coefficient \( d^V_{iVCD} \) are fixed their maximum values (HD) and with VCDs in the case of the proposed semi-active control (SA) respectively. The switching histories of the variable damping \( d^V_{iVCD} \) \((i = 1, 2, 3)\) in the semi-active control are shown in Fig. 5. The detailed results are summarized in Tables 2 and 3. In Table 2 the peak value of each absolute value of the response is presented. In Table 3 the performance is evaluated in the RMS value of each response. Note that the peak and the rms values of the force produced by \( i \)-th VCD are denoted by \( (f^i_{VCD}) \) in both tables. Firstly we can notice that a substantial performance improvement is accomplished when we install VCDs on the benchmark structure whenever the damping of each VCD is fixed or controlled. Furthermore the semi-active control shows almost better performance compared to the passive high damping case, i.e., the better vibration suppression property is achieved with relatively smaller force of VCDs. From this results we can claim the advantage of the proposed vibration control device and the semi-active control methodology.

## 5 CONCLUSION

In this paper we have studied the semi-active vibration control of civil structures with the new vibration control device (VCD). The proposed vibration control device can produce not only the damping force which is proportional to the relative velocity, but also the inertia force in proportion to the relative acceleration between two neighboring floors. Using the variable damping capability of VCD we propose a bang-bang type semi-active control which realizes the variable damping and the stiffness property of the structural systems based on Lyapunov theory. Moreover using the theory of dissipative dynamical systems (Willems, 1972; Kokotović and Arcak, 2001) we theoretically show that the semi-active control has a potential to improve the performance on vibration suppression compared to the high damping passive
### Table 1. Structural parameters.

<table>
<thead>
<tr>
<th>Structural parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>( m_1 = m_2 = m_3 )</td>
<td>6000 [kg]</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>( 1.7759 \times 10^3 ) [Ns/m]</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>( 3.6220 \times 10^3 ) [Ns/m]</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>( 4.4680 \times 10^3 ) [Ns/m]</td>
</tr>
<tr>
<td>( d_4 )</td>
<td>( -9.0279 \times 10^2 ) [Ns/m]</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>( 1.8475 \times 10^6 ) [N/m]</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>( 1.9152 \times 10^6 ) [N/m]</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>( 1.7453 \times 10^6 ) [N/m]</td>
</tr>
<tr>
<td>( k_4 )</td>
<td>( -1.8662 \times 10^5 ) [N/m]</td>
</tr>
</tbody>
</table>

Figure 3. Scaled El Centro NS wave (\(|\ddot{w}(t)| \leq 2.0[m/s^2]\)).

control. An appropriate selection of Lyapunov matrix is also discussed. With the simulation of NCREE benchmark building we conclude the effectiveness of the proposed hardware and the control scheme.

The experimental evaluation of the control system with real-scale structures is the future research subject.

### References


Figure 4. The absolute displacement and the acceleration of each floor.

Figure 5. The variable damping $d_{i}^{VCD}$ ($i = 1, 2, 3$).
Table 2. Performance of the semi-active control (Peak value evaluation).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Without VCD (NC&lt;sub&gt;peak&lt;/sub&gt;)</th>
<th>With VCD (High damping: HD&lt;sub&gt;peak&lt;/sub&gt;)</th>
<th>With VCD (Semi-active: SA&lt;sub&gt;peak&lt;/sub&gt;)</th>
<th>Δ&lt;sub&gt;peak&lt;/sub&gt; NC&lt;sub&gt;peak&lt;/sub&gt; (%)</th>
<th>Δ&lt;sub&gt;peak&lt;/sub&gt; HD&lt;sub&gt;peak&lt;/sub&gt; (%)</th>
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<td>54.92</td>
<td>14.02</td>
<td>13.46</td>
<td>24.51</td>
<td>96.02</td>
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<tr>
<td>q&lt;sub&gt;2&lt;/sub&gt; [mm]</td>
<td>97.04</td>
<td>22.14</td>
<td>20.67</td>
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<td>q&lt;sub&gt;3&lt;/sub&gt; [mm]</td>
<td>119.2</td>
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<tr>
<td>q&lt;sub&gt;1&lt;/sub&gt; [m/s&lt;sup&gt;2&lt;/sup&gt;]</td>
<td>4.309</td>
<td>1.101</td>
<td>1.063</td>
<td>24.67</td>
<td>96.58</td>
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<tr>
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<td>1.101</td>
<td>1.063</td>
<td>24.67</td>
<td>96.58</td>
</tr>
<tr>
<td>(f&lt;sub&gt;VCD&lt;/sub&gt;)&lt;sub&gt;1&lt;/sub&gt; [N]</td>
<td>–</td>
<td>2.027 × 10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>2.215 × 10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>–</td>
<td>109.3</td>
</tr>
<tr>
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<td>–</td>
<td>9.997 × 10&lt;sup&gt;3&lt;/sup&gt;</td>
<td>7.310 × 10&lt;sup&gt;3&lt;/sup&gt;</td>
<td>–</td>
<td>73.13</td>
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<tr>
<td>(f&lt;sub&gt;VCD&lt;/sub&gt;)&lt;sub&gt;3&lt;/sub&gt; [N]</td>
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<td>7.108 × 10&lt;sup&gt;3&lt;/sup&gt;</td>
<td>6.192 × 10&lt;sup&gt;3&lt;/sup&gt;</td>
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<td>87.11</td>
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</table>

Table 3. Performance of the semi-active control (RMS value evaluation).

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Without VCD (NC&lt;sub&gt;rms&lt;/sub&gt;)</th>
<th>With VCD (High damping: HD&lt;sub&gt;rms&lt;/sub&gt;)</th>
<th>With VCD (Semi-active: SA&lt;sub&gt;rms&lt;/sub&gt;)</th>
<th>Δ&lt;sub&gt;rms&lt;/sub&gt; NC&lt;sub&gt;rms&lt;/sub&gt; (%)</th>
<th>Δ&lt;sub&gt;rms&lt;/sub&gt; HD&lt;sub&gt;rms&lt;/sub&gt; (%)</th>
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<td>46.54</td>
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<td>10.42</td>
<td>22.33</td>
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<tr>
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<td>25.58</td>
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<td>–</td>
<td>2.027 × 10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>2.215 × 10&lt;sup&gt;4&lt;/sup&gt;</td>
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<td>6.192 × 10&lt;sup&gt;3&lt;/sup&gt;</td>
<td>–</td>
<td>87.11</td>
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