EXPERIMENTAL STUDY OF AN ADAPTIVE EXTENDED KALMAN FILTER FOR STRUCTURAL DAMAGE IDENTIFICATION

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ABSTRACT

An important objective of structural health monitoring systems is to identify the state of the structure and to detect the damage when it occurs. Analysis techniques for damage identification of structures, based on vibration data measured from sensors, have received considerable attention. Recently, a new adaptive tracking technique, based on the extended Kalman filter approach, has been proposed for the damage identification of structures. Simulation studies demonstrated that the adaptive extended Kalman filter (AEKF) approach is capable of tracking the variations of structural parameters, such as the degradation of stiffness, due to damages. In this paper, we present experimental studies to verify the capability of the AEKF approach in identifying the structural damage by conducting a series of experimental tests. A small-scale 3-story building model is used and the white noise excitations are applied to the top floor of the model. To simulate structural damages during the test, an innovative device is proposed to reduce the stiffness of some stories. Different damage scenarios have been simulated and tested. Measured response data and the AEKF approach are used to track the variation of stiffness during the test. The tracking results for stiffness are then compared with the stiffness predicted by the finite-element method. Experimental results demonstrate that the AEKF approach is capable of tracking the variation of structural parameters leading to the detection of structural damages.

Keywords: Experimental Study; System Identification and Damage Detection; Adaptive Extended Kalman Filter; Structural Health Monitoring

INTRODUCTION

An important objective of structural health monitoring systems is to identify the state of the structure and to detect the damage when it occurs. In this regard, analysis techniques for damage identification of structures, based on vibration data measured from sensors, have received considerable attention, and various approaches for system identification and damage detection have been proposed in the literature [e.g., Chang (1997, 1999, 2001)]. When a structural element is damaged, such as cracking, the stiffness of the damaged element is reduced. Hence, the structural damage may be reflected by the changes of parametric values of the damaged element. During a severe dynamic event, such as a strong earthquake, a structure may be damaged, and the damage event or the reduction of the

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stiffness of damaged elements will be contained in measured vibration data. To identify and track such structural damages, time domain analysis techniques may be used, including the least-square estimation (LSE) [Loh et al (2000); Lin et al (2001); Yang and Lin (2004, 2005a)], the extended Kalman filter (EKF) [Hoshiya and Saito (1984); Maruyama et al (1989); Sato and Takei (1998); Sato et al (2001); Yang et al (2006a, 2006b)], and sequential non-linear least-square estimation [Yang et al (2006c)]. Recently, a new adaptive tracking technique, based on the EKF approach, has been proposed for tracking the time-varying parameters on-line [Yang et al (2006a, 2006b)]. Simulation results demonstrate that this adaptive EKF approach (AEKF) is capable of tracking the variations of structural parameters, such as the degradation of stiffness, due to structural damages.

In this paper, experimental studies have been carried out to verify the capability of the AEKF approach in identifying the structural damage by conducting a series of experimental tests. For the experimental tests, a small-scale 3-story building model is used and a white noise excitation is applied to the top floor. To simulate structural damages during the test, an innovative device based on the concept presented in Yang et al (2000, 2005b, 2006d) is proposed and used to reduce the stiffness of some stories. Different damage scenarios have been simulated and tested. Measured response data and the AEKF approach are used to track the variation of story stiffness during the test. These experimental tracking results are then compared with the stiffness computed based on the finite-element method. It is demonstrated that the tracking results for the structural stiffness and their variations correlate well with that predicted by the finite-element analysis.

**ADAPTIVE EXTENDED KALMAN FILTER (AEKF)**

In this section, a brief summary of the newly proposed adaptive extended Kalman filter (AEKF) is given, and the details are referred to [Yang et al (2006a)]. Consider a m-DOF structure with the displacement vector, \( \mathbf{x} \), and velocity vector, \( \dot{\mathbf{x}} \). Let us introduce an extended state vector, \( \mathbf{Z}(t) = [\mathbf{x}^T, \dot{\mathbf{x}}^T, \mathbf{\theta}^T]^T \), where \( \mathbf{\theta}^T = [\theta_1, \theta_2, ..., \theta_n]^T \) is an \( n \)-unknown parametric vector with \( \theta_i \) \((i = 1, 2, ..., n) \) being the \( i \)th unknown parameter of the structure, including damping, stiffness, nonlinear and hysteretic parameters. The equation of motion of the structure can be expressed as

\[
d\mathbf{Z}(t)/dt = \mathbf{g}(\mathbf{Z}, \mathbf{f}, t) + \mathbf{w}(t) \tag{1}
\]

in which \( \mathbf{w}(t) \) is model noise (uncertainty) with zero mean and a covariance matrix \( \mathbf{Q}(t) \), and \( \mathbf{f} \) is the excitation vector. A nonlinear discrete equation for an observation vector (measured output) can be expressed as follows,

\[
\mathbf{Y}_{k+1} = \mathbf{h}(\mathbf{Z}_{k+1}, \mathbf{f}_{k+1}, k + 1) + \mathbf{v}_{k+1} \tag{2}
\]

in which \( \mathbf{Y}_{k+1} \) is a \( l \)-dimensional observation (measured) vector at \( t = (k + 1)\Delta t \) (sampling time step \( \Delta t \)), i.e., \( \mathbf{Y}_{k+1} = \mathbf{Y}(t = (k + 1)\Delta t) \), \( \mathbf{Z}_{k+1} = \mathbf{Z}(t = (k + 1)\Delta t) \), and \( \mathbf{f}_{k+1} = \mathbf{f}(t = (k + 1)\Delta t) \). In Eq.(2), \( \mathbf{v}_{k+1} \) is a measurement noise vector assumed to be a Gaussian white noise vector with zero mean and a covariance matrix \( \mathbf{E}[\mathbf{v}_k\mathbf{v}_k^T] = \mathbf{R}_k \delta_{kj} \) where \( \delta_{kj} \) is the Kroneker delta.

Let \( \hat{\mathbf{Z}}_{k+1|k} \) be the estimate of \( \mathbf{Z}_{k+1} \) at \( t = (k + 1)\Delta t \), and \( \hat{\mathbf{Z}}_{k+1|k} \) be the estimate of \( \mathbf{Z}_{k+1} \) at \( t = k\Delta t \). Based on the AEKF, the recursive solution for the estimate \( \hat{\mathbf{Z}}_{k+1|k+1} \) of the extended state vector is given by

\[
\hat{\mathbf{Z}}_{k+1|k+1} = \hat{\mathbf{Z}}_{k+1|k} + \mathbf{K}_{k+1}[\mathbf{Y}_{k+1} - \mathbf{h}(\hat{\mathbf{Z}}_{k+1|k}, \mathbf{f}_{k+1}, k + 1)] \tag{3}
\]

\[
\hat{\mathbf{Z}}_{k+1|k} = \mathbf{E}\{\mathbf{Z}_{k+1|k} | \mathbf{Y}_1, \mathbf{Y}_2, ..., \mathbf{Y}_k\} = \hat{\mathbf{Z}}_{k|k} + \int_{k\Delta t}^{(k+1)\Delta t} \mathbf{g}(\mathbf{Z}_t|k, \mathbf{f}, t) dt \tag{4}
\]
in which \( K_{k+1} \) is the Kalman gain matrix

\[
K_{k+1} = P_{k+1|k} H_{k+1|k}^T [H_{k+1|k} P_{k+1|k} H_{k+1|k}^T + R_{k+1}]^{-1}
\]

(5)

In Eq.(5), \( P_{k+1|k} \) and \( H_{k+1|k} \) are given by

\[
P_{k+1|k} = \Lambda_{k+1} [\Phi_{k+1,k} P_{k|k} \Phi_{k+1,k}^T] \Lambda_{k+1}^T + Q_{k+1}
\]

(6)

\[
H_{k+1|k} = [\partial h(Z_{k+1}, f_{k+1,k} + 1)/\partial Z_{k+1}] Z_{k+1} = \dot{Z}_{k+1|k}
\]

(7)

\[
P_{k|k} = [I_{2m+n} - K_k H_{k|k-1}] P_{k|k-1} [I_{2m+n} - K_k H_{k|k-1}]^T + K_k R_k K_k^T
\]

(8)

In Eq.(6), \( \Lambda_{k+1} \) is a diagonal matrix, referred to as the adaptive factor matrix. The determination of \( \Lambda_{k+1} \) has been described in [Yang, et al (2006a)]. In the recursive solution above, \( P_{k|k} \) is the error covariance matrix of the estimated extended state vector. To initiate the recursive solution, the initial values for the unknown parameters and unknown state vector should be assumed. Likewise the initial error covariance matrix \( P_{0|0} \) of the estimated extended state vector, the covariance matrix \( R \) of the measurement noise vector \( v(t) \), and the covariance matrix \( Q \) of the system noise vector \( w(t) \) should be assumed.

### EXPERIMENTAL STUDY

#### Experimental Set-Up

A 400 mm by 300 mm small-scale 3-story building model, as shown in Fig. 1, is used in the experiment. The height of this building model is 885 mm and the total weight is 75.4 kg. The mass of each floor is 24.4 kg, and the stiffness of each story is obtained as 55.5 kN/m using the finite-element model. Based on the discretized 3-DOF shear-beam model, the first three natural frequencies are: 3.38 Hz, 9.47 Hz, and 23.68 Hz, respectively. A white noise excitation force is applied to the top floor in one direction using an exciter equipped with a force sensor (PCB2008C03). Each floor is installed with one acceleration sensor and one displacement sensor to measure the floor responses.

The damage in a story unit is assumed to be reflected by the reduction of its stiffness. To simulate the reduction of stiffness in a selected story unit, say \( i \) th story, we install a stiffness element device (SED) with an effective stiffness of \( K_{hi} \) in the \( i \) th story unit, so that the stiffness of the \( i \) th story unit is increased by \( K_{hi} \). During the experimental test, the effective stiffness of the SED is reduced to zero to simulate the reduction of the stiffness in the \( i \) th story unit. This innovative concept for the stiffness element device (SED) is motivated by the so-called resetable semi-active stiffness dampers [e.g., Yang et al (2000, 2005b, 2006d)] as described in the following.

Let us consider a device consisting of a hydraulic cylinder-piston (HCP) system with one valve on each side of the piston as shown in Fig. 2. The cylinder is filled with pressurized gas. When both valves are closed, the HCP serves as a stiffness element in which the stiffness is provided by the bulk modulus of the pressurized gas in the cylinder. When both valves are open, the piston is free to move and hence the stiffness of the HCP is zero. For simulating the stiffness reduction in a selected story unit, the HCP is connected to a bracing system and installed in the selected story unit as shown in Fig. 3. In Fig. 3, the HCP (hydraulic cylinder-piston) is fixed to the bracing system in the first story, and the piston is connected to the first floor. Hence, the HCP and the bracing system are connected in series. The entire system, consisting of the HCP and the bracing system is referred to as the stiffness element device (SED).

Suppose the stiffness of the HCP is denoted by \( K_{hi} \) and that of the bracing system in the \( i \) th story is denoted by \( K_{bi} \). Then, the effective stiffness of the entire SED, denoted by \( K_{hi} \) is given by
K_{hi} = \frac{K_{ei} K_{bi}}{K_{ei} + K_{bi}} \quad (9)

since both $K_{ei}$ and $K_{bi}$ are connected in series. In our experimental set-up, the stiffness of the bracing system $K_{bi}$ is much bigger than that of the HCP, i.e., $K_{bi} \gg K_{ei}$. Hence, the effective stiffness of the entire SED, consisting of the HCP and the bracing system, is approximately equal to that of the HCP, i.e., $K_{hi} = K_{ei}$. With the installation of the SED in the first story as shown in Fig. 3, the stiffness of the first story is increased by $K_{hi}$.

The stiffness of the HCP, $K_{ei}$, depends on the magnitude of the gas pressure in the cylinder. It has been shown in Yang et al (2005b) that $K_{ei}$ is linearly proportional to the gas pressure $P_0$, i.e.,

$$K_{ei} = G P_0 \quad (10)$$

where $G$ is a constant depending on the dimension of the cylinder and the property of the fluid or gas. Consequently, the desirable effective stiffness of the SED, $K_{hei}$, can be achieved by adjusting the gas pressure $P_0$ in the cylinder.
To simulate the reduction of stiffness in the \( i \)th story during the experimental test, a SED system is installed in the \( i \)th story, so that the stiffness of the \( i \)th story is increased by \( K_{hi} = K_{ei} \). During the experimental test, two valves are open simultaneously at the time instant, \( t_r \), so that \( K_{hi} \) becomes zero, thus reducing the stiffness of the \( i \)th story by an amount of \( K_{ei} \) at \( t = t_r \). The installation of a SED system in the second story of the test model is shown in Fig. 4.

**EXPERIMENTAL RESULTS**

To demonstrate the capability and accuracy of the adaptive extended Kalman filter (AEKF) approach for tracking the structural damage on-line or almost on-line [Yang et al (2006a)], experimental tests are conducted herein for different damage scenarios. Due to space limitation, only three damage cases are presented in the following.

**Case 1: A Damage in First Story**

In this test, a stiffness element device (SED), consisting of a hydraulic cylinder-piston (HCP) and a bracing system, is installed in the first story unit as shown in Fig. 3. The cylinder is filled with air at an air pressure of 0.4 MPa. From the experimental test of the SED, a gas pressure at \( P_0 = 0.4 \) MPa results in an effective stiffness of 6.0 kN/m, i.e., \( K_{hi} = K_{ei} = 6.0 \) kN/m. Thus, the stiffness of the first story is \( k_1 = 55.5 \) kN/m + 6.0 kN/m = 61.5 kN/m, whereas the stiffness of other two stories is \( k_2 = k_3 = 55.5 \) kN/m.

A band-limited white-noise excitation (force) in the frequency range of 0-25 Hz was applied to the top floor of the building model. During the test, both valves of the SED system were open simultaneously at \( t = 25 \) seconds, so that the stiffness in the first story reduces abruptly from 61.5 kN/m to 55.5 kN/m at \( t = 25 \) seconds. The white-noise excitation force, \( F \), and the acceleration responses of all floors, \( a_1, a_2 \) and \( a_3 \), were measured and presented in Fig. 5. The sampling frequency of all measurements is 200 Hz. Further, the displacement responses of all floors were also measured for correlation studies.

With the measured excitation and acceleration responses shown in Fig. 5, the unknown parameters for the stiffness and damping of all stories, i.e., \( k_i \) and \( c_i \) (\( i = 1, 2, 3 \)), can be identified on-line using the AEKF approach [Yang et al (2006a)]. For the AEKF approach described previously, the following initial values were assumed: (i) the initial values for \( k_i \) and \( c_i \) are: \( k_{i0} = 40 \) kN/m and \( c_{i0} = 0.1 \) kN.s/m (\( i = 1, 2, 3 \)), (ii) The initial values for the displacements and velocities are zero, i.e., \( \dot{x} = 0, \ddot{x} = 0 \), (iii) the initial error covariance matrix \( P_{0|0} \) of the extended state vector is a \((12\times12)\) diagonal matrix with the first 6 diagonal elements being 0.1 and the last 6 diagonal elements being 100, and (iv) The covariance matrices of the measurement noise vector \( \nu(t) \) and the system noise vector \( \omega(t) \) are chosen to be \( R = 0.1I_3 \) and \( Q = 10^{-9} I_{12} \), respectively, where \( I_j \) is a \((j\times j)\) unit matrix. In all the experimental studies to be presented later, these initial values will be used.

Based on the AEKF tracking technique and the measured data shown in Fig. 5, the identified stiffness parameters for all stories are presented in Figure 6(a) as solid curves. Also shown in Fig. 6(a) as dashed curves for comparison are the results based on the finite-element computation. Further, the identified displacements of all floors using AEKF are presented in Fig. 6(b) as solid curves, whereas the dashed curves shown in Fig. 6(b) are the measured experimental results. In Fig. 6(b), both solid curves and dashed curves coincide, indicating that the accuracy of the AEKF approach is excellent. It is observed from Fig. 6(a) that the identified stiffness based on AEKF (solid curves) is reasonably close to that predicted by the finite element method (dashed curves). The difference between the solid and dashed curves has been expected due to the uncertainty of the test model. Fig. 6(a) clearly demonstrates that the AEKF approach is capable of tracking the variation of stiffness parameters, leading to the detection of structural damages.
Case 2. A Damage in Second Story

In this test, a SED system is installed in the second story, i.e., between the first and second floors, as shown in Fig. 4. The total weight of the SED system is 5.1 kg, which should be added to the first floor. Hence, the mass of the first floor is $m_1 = 29.5$ kg, whereas the mass of other floors is $m_2 = m_3 = 24.4$ kg. The air pressure $P_0$ in the cylinder is 0.7 MPa. Based on the experimental tests, the stiffness of the SED with an air pressure of 0.7 MPa is 10.5 kN/m, which should be added to the second story. Thus, the stiffness of the second story is $k_2 = 66$ kN/m, whereas the stiffness of other stories is $k_1 = k_3 = 55.5$ kN/m prior to the test. Band-limited white-noise excitations were applied to the top floor in the experimental test. During the test, both valves in the SED were open at the time instant $t = 18$ seconds, so that the stiffness of the second story is reduced abruptly from 66 kN/m to 55.5 kN/m. Absolute accelerations of all floors, $a_1$, $a_2$ and $a_3$, and the excitation force $F$ are measured and shown in Fig. 7. The displacement responses of all floors are also measured and shown in Fig. 8(b) as dashed curves.
Again, the sampling frequency for all measured signals is 200Hz. Unknown parameters to be identified are $k_i$ and $c_i$ ($i = 1, 2, 3$).

Based on the experimental data shown in Fig.7 and the adaptive extended Kalman filter (AEKF), the identified stiffness for all stories, i.e., $k_i$ ($i = 1, 2, 3$) are presented in Fig.8(a) as solid curves. Also shown in Fig. 8(a) as dashed curves for comparison are the stiffness results estimated using the finite-element method. The predicted displacement responses of all floors based on AEKF are presented in Fig. 8(b) as solid curves. In Fig. 8(b), the solid curves and dashed curves (measured experimental data) almost coincide, indicating the excellent accuracy of the AEKF approach in predicting the displacement responses. Further, Fig. 8(a) demonstrates that the AEKF approach is capable of tracking the variation of stiffness leading to the identification of structural damages.

![Figure 7: Measured time-histories of acceleration responses and excitation force; Case 2.](image)

![Figure 8: Identified parameters based on AEKF (Case 2); (a) identified stiffness, and (b) identified displacements.](image)
Case 3. Damages in First and Second Stories

In this test, one SED system is installed in the first story and another SED system is installed in the second story to simulate the damages in both stories. The air pressures in both SED systems are identical, i.e., \( P_0 = 0.7 \, \text{MPa} \), so that the effective stiffness of each SED is \( K_{hi} = 10.5 \, \text{kN/m} \). Hence, prior to the test, we have \( k_1 = k_2 = 66.0 \, \text{kN/m} \) and \( k_3 = 55.5 \, \text{kN/m} \). Again, white-noise excitations were applied to the top floor. During the test, valves of the SED system in the first story were open at \( t = 35 \) seconds and that of the SED system in the second story were open at \( t = 27 \) seconds. Acceleration responses of all floors and the excitation force were measured and shown in Fig. 9. The displacement responses of all floors were also measured and shown in Fig. 10(b) as dashed curves. The sampling frequency for all the measured data is 200 Hz.

Based on the measured experimental data shown in Fig. 9 and the AEKF approach, the identified stiffness for all stories, i.e., \( k_i \) \((i = 1, 2, 3)\) are presented in Fig. 10(a) as solid curves. The dashed curves shown in Fig. 10(a) for comparison are the stiffness estimated using the finite-element method. The AEKF predicted displacement responses are presented in Fig. 10(b) as solid curves, which coincide with the measured experimental results shown by dashed curves. It is observed from Fig. 10 that the adaptive extended Kalman filter (AEKF) approach is capable of tracking the variation of structural stiffness quite well.

From Figs. 6(a), 8(a) and 10(a), the following conclusions are obtained: (i) the stiffness \( k_1 \) of the first story predicted by AEKF is almost identical to that estimated by the finite-element method, (ii) the stiffness \( k_2 \) of the second story predicted by AEKF is uniformly smaller than that estimated by the finite-element method, and (iii) the stiffness \( k_3 \) of the third story predicted by AEKF is uniformly bigger than that estimated by the finite-element method. The trend above is consistent for all the test results, i.e., Figs. 6(a), 8(a) and 10(a). Such discrepancies have been expected due to the model uncertainty. Finally, it is quite consistent that the displacement responses predicted by AEKF correlate very well with that of the measured experimental data, as demonstrated in Figs. 6(b), 8(b) and 10(b).

![Figure 9: Measured time-histories of acceleration responses and excitation force; Case 3.](image)
CONCLUSIONS

Recently, a new adaptive extended Kalman filter (AEKF) has been proposed for the damage identification of structures. In this paper, we have performed experimental studies to verify the capability of this AEKF in identifying the structural damage by conducting a series of experimental tests on a scaled 3-story building model. To simulate the structural damage during the test, an innovative stiffness element device has been proposed to reduce the stiffness of some building stories. Different damage scenarios have been simulated and tested. Measured response data and the AEKF have been used to track the variation of the stiffness of different stories during the test. The identified stiffness parameters based on the AEKF correlate very well with that predicted by the finite-element method. Experimental results demonstrate that the AEKF approach is capable of tracking the variation of stiffness parameters leading to the detection of structural damages.

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