3-D NONLINEAR ANALYSIS OF SOIL-PILE INTERACTION IN LIQUEFIABLE SOIL USING ADAPTIVE MESH REFINEMENT

Xiaowei Tang¹ Yuan Di² Maotian Luan³ Sumio Sawada⁴ and Tadanobu Sato⁵

ABSTRACT

In 3-D finite element analysis of pile behavior in liquefiable soil during an earthquake, especially considering large deformation of liquefied ground, error due to discretization in the zone near the pile becomes very large. To improve the accuracy and increase the efficiency of finite element analysis, an h-adaptive scheme that included a posteriori error estimation and mesh refinement was applied to the analysis. The calculated results of effective stress were smoothed locally by superconvergent patch recovery method and smoothed strain was used to calculate the L₂ norm of the strain error in the last step of the calculation of each time increment. The mesh was refined by a fission procedure belonging to h-refinement based on the indication of the error estimate. As a numerical example, a soil-pile interaction system loaded cyclically was analyzed by our method.

Keywords: Adaptive mesh refinement, Soil-pile interaction, Liquefaction

INTRODUCTION

During strong earthquakes liquefaction leads to large deformation of the liquefied ground and damages foundations, in particular, pile foundations. Investigations of the 1964 Niigata and 1995 Hyogoken-Nanbu earthquakes showed a prevalence of such failures. Since then, nonlinear analysis of the liquefaction phenomenon by FEM has been conducted in the fields of soil-pile interaction and pile foundation design. Owing to large deformation caused by liquefaction, the finite deformation theory (FDT) is more suitable than the normal one that is based on the assumption of small deformation, therefore the FDT has been used in the analysis of liquefaction by the finite element method. Although it effectively deals with the geometrical non-linearity of liquefied soil, it has several problems. As a type of numerical approximation, errors are inevitable in the analysis results obtained. Moreover the finite element method solution does not always produce the desired accuracy and sometimes causes serious analysis problems. For example, in liquefaction analysis of saturated soil considering large deformation, when a coarse mesh is used to save time, error causes severe distortion of the elements, and sometimes calculations stop unexpectedly. The accuracy of the finite element method is still a major concern of numerical analysis, particularly when a non-linear material response is occurred and large deformation is involved.

¹ Associate Professor, Institute of Geotechnical Engineering, School of Civil and Hydraulic Engineering Dalian University of Technology, Dalian, China; Researcher, Disaster Prevention Research Institute, Kyoto University, Kyoto, Japan.
² Assistant Professor, College of Engineering, Peking University, Beijing, China.
³ Professor, Institute of Geotechnical Engineering, School of Civil and Hydraulic Engineering, Dalian University of Technology, Dalian, China.
⁴ Professor, Disaster Prevention Research Institute, Kyoto University, Kyoto, Japan.
⁵ Professor, School of Science and Engineering, Waseda University, Tokyo, Japan.
In soil-pile interaction analysis, errors involving elements surrounding piles are very large. This error in the finite element method is caused by discretization. Evidently, reducing element size uniformly during discretization minimizes error, but the number of nodes and elements are increased as is the calculation time. Our objective was to use a fine mesh in the area of large error and a normal or coarse mesh in that of low error. A method, called the adaptive technique or adaptive mesh refinement, has been developed and used to solve this error problem. An adaptive finite element method, in which approximation is refined successively to obtain a predetermined standard of accuracy, is essential for the effective use of finite element codes in practical analyses. This procedure, which refines the mesh of the finite elements according to an error indicator, has two parts: error estimation and mesh refinement. In error estimation, error is defined as the difference between the approximate and exact solutions of such variables as displacement, stress, and strain. Generally, it is estimated by means of the energy norm or $L_2$ norm. $H$-version mesh refinement is the simple reduction of subdivision size, including remeshing and fission.

We applied the $h$-adaptive finite element method to liquefaction analysis of saturated soil considering large deformation in a soil-pile interaction problem. Linear, eight-node cubic elements were used in the discretization. A posteriori error estimate procedure based on evaluating the $L_2$-norm of variable error and the superconvergent patch recovery (SPR) procedure (Zienkiewicz and Zhu, 1992) was used in this study. A fission procedure belonging to $h$-refinement (Belytschko and Wong, 1989) was adopted for the mesh refinement of soil elements. After one calculation step, elements which exceed a given error limit are fissioned into 8 elements, and the next step executed. The seismic response of a concrete pile resting on saturated sand was analyzed, and the elements of liquefied sand were well refined by our method. The efficacy of this technique for liquefiable soil analysis is shown.

**GOVERNING EQUATIONS USING UPDATED LAGRANGIAN FORMULATION**

The two-phase mixture theory was used in the analysis of liquefaction. It was proposed by Biot and has been widely used in non-linear analysis of saturated soils. Using it and neglecting acceleration of the pore fluid, the equilibrium equation for saturated soil was obtained. With the mass conservation equation for fluid flow and assuming that the porosity distribution of the medium is sufficiently smooth and that solid skeletons are incompressible, the initial strain rate is 0, the simplified equation for fluid flow and assuming that the porosity distribution of the medium is sufficiently smooth and that solid skeletons are incompressible, the initial strain rate is 0, then the simplified continuity equation is derived. Integrating the equilibrium equation in the spatial domain by the finite difference method, the governing equations of the mixed FE-FD method with $u$-$p$ formulations based on the assumption of infinitesimal deformation (Akai and Tamura, 1978).

To deal with the large deformation of liquefied soil, a new version of this scheme (Di and Sato 2001) was used, in which the updated Lagrangian formulation was adopted. Derivation of the governing equations was introduced. In that formulation, all variables at time $t$ are taken as the reference configuration of variables at time $t+\Delta t$. The reference configuration is updated at each calculation step. Rayleigh damping also is added to the governing equations, and the Newmark-$\beta$ method for time domain integration used to solve the dynamic equations. The last governing equations in $u$-$p$ form obtained, as shown in Eq.(1), is

$$
\begin{align*}
\alpha^\Delta t & \sum_i \left( M + \gamma \Delta t \frac{\partial}{\partial t} C + \beta \Delta t \frac{2+\beta}{2} (K_e + K_{nl}) \right) \frac{\partial^2 u_i}{\partial t^2} = \\
& - \left( H + \frac{\partial}{\partial t} L \right) \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial t} Q \cdot \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial t} p \cdot \frac{\partial u_i}{\partial t} - \frac{\partial}{\partial t} Q \left( \frac{\partial u_i}{\partial t} + (1-\gamma) \Delta t \frac{\partial^2 u_i}{\partial t^2} \right) \left( \frac{\partial^2 u_i}{\partial t^2} + \Delta \frac{\partial u_i}{\partial t} \right) \\
& + \frac{\gamma}{k} \frac{\partial}{\partial t} Q \cdot \left( \frac{\partial u_i}{\partial t} + (1-\gamma) \Delta t \frac{\partial^2 u_i}{\partial t^2} \right) + \frac{\partial}{\partial t} L_T \cdot \frac{\partial^2 u_i}{\partial t^2} \\
& \text{where the mass matrix } M = \int_V \rho N^T N dV; \text{ the stiffness matrix } K_e = \int_V B_e^T (D + \psi \mathbf{B}_e) B_e dV; \text{ the geometric stiffness matrix } K_{nl} = \int_V B_{nl}^T A B_{nl} dV; \text{ and} \\
& Q = -\int_V B_e^T dV; T = \int_V B_e^T F - \int_V B_e^T \sigma dV; \alpha = \frac{\gamma}{g} \frac{\rho}{k} \frac{\partial}{\partial t} \\
& \text{H}_p = \rho_e \sum_{m} (n_e \cdot n_e) \mathbf{b}_m + \sum_{m} \rho_e (n_e \cdot n_e) \mathbf{b}_m; \mathbf{L}_p = \int_V \frac{\gamma}{k} \frac{\partial}{\partial t} \mathbf{p}_e dV
\end{align*}
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The reference configuration is updated at each calculation step. Rayleigh damping also is added to the governing equations, and the Newmark-$\beta$ method for time domain integration used to solve the dynamic equations. The last governing equations in $u$-$p$ form obtained, as shown in Eq.(1), is

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& - \left( H + \frac{\partial}{\partial t} L \right) \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial t} Q \cdot \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial t} p \cdot \frac{\partial u_i}{\partial t} - \frac{\partial}{\partial t} Q \left( \frac{\partial u_i}{\partial t} + (1-\gamma) \Delta t \frac{\partial^2 u_i}{\partial t^2} \right) \left( \frac{\partial^2 u_i}{\partial t^2} + \Delta \frac{\partial u_i}{\partial t} \right) \\
& + \frac{\gamma}{k} \frac{\partial}{\partial t} Q \cdot \left( \frac{\partial u_i}{\partial t} + (1-\gamma) \Delta t \frac{\partial^2 u_i}{\partial t^2} \right) + \frac{\partial}{\partial t} L_T \cdot \frac{\partial^2 u_i}{\partial t^2} \\
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& \text{H}_p = \rho_e \sum_{m} (n_e \cdot n_e) \mathbf{b}_m + \sum_{m} \rho_e (n_e \cdot n_e) \mathbf{b}_m; \mathbf{L}_p = \int_V \frac{\gamma}{k} \frac{\partial}{\partial t} \mathbf{p}_e dV
\end{align*}
$$
In these equations, $C$ is the Rayleigh damping matrix; $?'$ and $?^\beta$ are parameters of the Newmark-$\beta$ method; $?^\rho$ is the density of the soil; $?^\rho f$ the weight of the pore fluid per unit volume; $n$ the porosity, $k$ the permeability coefficient; $pE$ the excess pore pressure value at the gravity center of an element; $pE_i$ that of an adjacent element; $A_i$ the area of the joint surface between an element and an adjacent one, $i$; $n_i^A$ the normal direction vector of $A_i$; $n^E_i$ the normal direction vector of dissipation from the element to its adjacent element, $i$; and $n^E_{i,j}$ the normal direction vector of dissipation from the adjacent element, $i$, to the element.

An effective cyclic elasto-plastic constitutive model (Oka et al., 1994) based on the effective stress criterion and kinematic hardening rule is used to simulate the non-linear behavior of saturated soil. The stress-dilatancy relationship and cumulative strain-dependent characteristics of the plastic shear modulus are taken into account. Simulation results for saturated soil agree with experimental results, even those for the liquefaction process. This constitutive model is incorporated in a mixed FE-FD coupled method.

**ADAPTIVE MESH REFINEMENT**

An adaptive FE method, in which approximation is refined successively to reach a predetermined standard of accuracy, is essential for the effective use of finite element codes in practical analyses. The procedure, which refines the mesh of the finite elements according to an error indicator, has two parts: error estimate and mesh refinement.

**A posteriori Error Estimate**

A posteriori error estimate procedure based on evaluating the $L^2$-norm of strain error and the superconvergent patch recovery (SPR) procedure was used in this study. The formulations were derived for linear hexahedral elements. Error is defined as the difference between the exact solution and value of the finite element approximation. Variables considered in the error estimate are displacement, strain, and stress. For strain, an error is described as

$$ e^e = \varepsilon^e - \varepsilon^b $$

In the practical adaptivity process, a relative percentage error generally is used because it is more easily interpreted. The relative percentage error of solution is estimated using the error norm and the strain $L^2$-norm of entire solution domain. For $i$-th element, its definition is

$$ \eta = \frac{\left\| e^e \right\|}{\sqrt{\left\| e^e \right\|^2 + \left\| e^b \right\|^2}} \times 100\% $$

In the error estimate process, the relatively accurate values rather than the exact solution are used to calculate errors because the exact solution is impossible to obtain for usual complex systems. The superconvergent patch recovery technique was used. It gets smoothed value by solving Eq. (4).

$$ \sum_{i=1}^{V} \left\{ P^T(x_i, y_i) \varepsilon(x_i, y_i) \right\}^{T} = \sum_{i=1}^{V} \varepsilon^b(x_i, y_i) \left\{ P^T(x_i, y_i) \right\}^{T} $$

**$H$-adaptive Mesh Refinement**

In three-dimension, linear hexahedra elements are used in adaptive finite element analysis. When the error of an element exceeds an acceptable limit, the element is fissioned into eight children elements. When an element is fissioned next to an unfissioned one, slave nodes are created, and they are constrained by the compatibility condition of master node. Instead the nodal forces at the slave nodes are added to the forces at the corresponding master nodes; for the analysis of soil-pile interaction adaptive analysis, in the joint surface between soil and pile, the effect of relative deformations of pile elements to soil elements is so small that we almost can neglect it because concrete is much harder than soil. For this reason, the difference of fission level between soil elements and pile element is allowed to be more than 1 in the adaptive analysis of this research, shown as Figure1.
NUMERICAL EXAMPLE

We use the soil-pile interaction model to test the efficacy of our method as shown in Figure 2. A concrete pile is shown by the shadow elements, which is driven in 8m-deep saturated Edosaki sand (Dr=40%) and 4m-deep silica. The Yang’s modulus of the pile is E=2.45e7 kN/m². A 400kN mass is added on top of pile as the load from super structure. The selected area of the soil is 42m long and 16m wide, and the diameter of the pile is 1.2m. The pile is resting in the center. The constitutive model of soil is an effective cyclic elasto-plastic model. The elements of soil are linear eight-node hexahedral elements with 2m length. The pile is described by an elastic column model. The displacement of two sides boundaries is allowed in the vertical direction only. The fixed boundary condition is given for the bottom nodes. Drainage is only allowed on the top of the soil. Two load cases are analyzed with the adaptive FE method. As shown in Figure 2, the first one is horizontal force acting on the top of pile. The value of the force increases gradually to 100 kN during 2 seconds. It is used to demonstrate the error estimates and adaptive FE method. Another load is an acceleration time history as shown in Figure 2. It is used to check the efficacy of adaptive FE method applied to three-dimensional seismic analysis of soil-pile interaction.

Only soil elements are refined because that large deformation of soil elements makes relative error larger. The refinement of these elements is effective to improve the accuracy of soil-pile interaction analysis. The initial mesh of adaptive FE analysis is a coarse mesh with 330 elements.
The relative error contour at the time $t=1.0$ second is given in Figure 3. The refinement starts from time $t=1.0$ second according to this distribution of the error. The relative error of the elements surrounding the head of the pile is large because the large displacement of pile causes the large deformation of surrounding soil elements as shown in the figure.

The error limit of $h$-adaptive refinement is 0.05. The elements with the relative error larger than the limit are fissioned. Two adaptive frequencies are used in this adaptive analysis. In Figure 4, the refined meshes at time $t=2.0$ second are given. The mesh is refined one time per 0.5 second and 0.2 second respectively. The refinement level is two. Any initial element is not allowed to be refined more than twice. The elements, whose relative error values as shown in Figure 3 are larger than the limit, are refined well step by step. The element numbers of refined mesh with $dt=0.2$ second are more than those with $dt=0.5$ second in Figure 4. It shows that higher refinement frequency cases finer quality of mesh.

In Figure 5, the values of average relative error calculated in three different cases are compared. The two error values of adaptive analysis becomes lower than the value of fixed coarse mesh after adaptive process starts at time $t=1.0$ second. The efficiency of adaptive FE method applied to three-dimensional soil-pile interaction analysis is demonstrated. The adaptive analysis with high refinement frequency gives a quick and stable improvement of accuracy as shown in the figure. At the same time, more elements are created. In the refinement of three-dimensional mesh, one element fission to eight elements. The increase speed of element number is very fast than the speed of two-dimensional adaptive analysis.

In three-dimensional analysis of soil, high level of refinement can not give further evident improvement of accuracy but cases heavy burden of calculation. With improving the accuracy, it is necessary to control the calculation work via giving the limit of mesh refinement level and reducing the frequency of mesh refinement.

The example of three-dimensional soil-pile interaction analysis with a monotonous load demonstrates the efficacy of adaptive FE method. The method improves the accuracy of the mesh effectively by refining the soil elements with large error surrounding the head of pile. The error estimator can find these elements effectively.
Result of seismic analysis of 3-D soil-pile interaction

The second case is 3-D seismic analysis of soil-pile interaction with adaptive FE method. The input acceleration history is shown in Figure 6 with a maximum value of 722 cm/sec². For comparison, responses without adaptive process are also calculated with the fixed coarse mesh with and the fixed fine mesh.

The analysis result of the fixed coarse mesh (330 elements) and the fixed fine mesh (2268 elements) without adaptive process and their relative error contours are given in Figure 7. The relative error values of upper elements are large especially the elements surrounding the head of the pile. In general, the relative error of the fine mesh is smaller than that of the coarse mesh.

In adaptive analysis, the limit of relative error is 0.07. The mesh refinement starts from time $t=5.5$ second and is carried out one time every 0.5 second. The number of limit for the refinement is 2. The refined mesh at time $t=15.0$ second are shown as Figure 8. It is easy to find that the elements in the region with large error shown in Figure 7 are refined well. The element number increases quickly with the mesh refinement.

![Figure 7. Results of fixed coarse and fine meshes and relative error for 3-D soil-pile interaction](image)

![Figure 8. Refined meshes of 3-D soil-pile interaction](image)

The final horizontal displacements of the pile calculated in the three cases are compared in Fig. 9. The pile deformation of the fixed coarse mesh is larger than that of the fixed fine mesh. The adaptive refinement of soil elements improves the accuracy of the coarse mesh and makes the horizontal displacement to approach the value of the fine mesh. Figure 10.(a) shows the horizontal displacement curves of pile head. The displacement of coarse mesh is larger than that of the fine mesh. With improvement the accuracy of the mesh using adaptive FE method, the displacement approaches to the result of fine mesh evidently. In a usual way, the average relative errors of the three cases are compared between the results of adaptive procedure and the results of the finite element analysis with

![Figure 9. Displacements of pile(t=15 sec)](image)
the fixed coarse mesh and the fixed fine mesh in Figure 10.(b) too. The error value of the coarse mesh is larger than the value of the fine mesh. In adaptive FE analysis, before the adaptive process starts, the error value is same as the value of the initial coarse mesh. The adaptive mesh refinement lowers the error value to reach a new lower value.

The results of these two examples demonstrate the efficiency of h-adaptive FE method applied to three-dimensional soil-pile interaction analysis. With this method, the soil elements in large deformation are refined well and the accuracy of analysis is improved effectively.

Figure 10. (a) Horizontal displacement of pile head; (b) average relative error of soil.

CONCLUSIONS

The h-adaptive finite element method was used in the nonlinear analysis of soil-pile interaction considering large deformation caused by soil liquefaction. The adaptive procedure was used only for soil elements in order to analyse the liquefaction of saturated sand. The approximation was refined successively to satisfy a predetermined standard of accuracy, and its efficacy confirmed by finite element analysis. This method is easy to use to solve practical engineering problems.

The a posteriori error estimate based on the L^2 norm of strain or stress error was adopted. It effectively estimates elements error after each calculation step in the nonlinear finite element analysis of soil. The superconvergent patch recovery technique was used to get smoothed value for error estimation. Calculations based on this method can easily implement any code and clearly are advantageous for saving computation time. The efficacy of this error estimator in the dynamic analysis of porous media was shown by a simple example to be a reliable error indicator for mesh refinement.

The program developed for soil-pile interaction that includes liquefaction analysis was modified by means of this indicator. A numerical example was given to show the efficacy of our method. The saturated sand of a soil-pile interaction system was analysed, including the liquefaction process. The findings show that this adaptive scheme provides substantial improvement in accuracy with minimal additional computation. Generally, an adaptive mesh provides a one order-higher level of accuracy than that obtained by a fixed mesh with less than half the computational resources.

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REFERENCES


